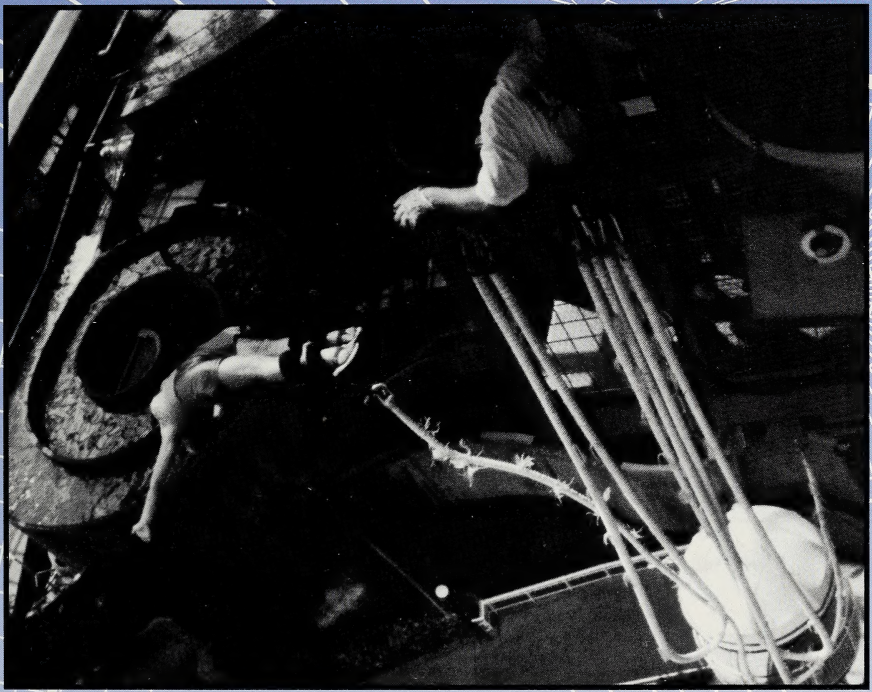
PHYSICS 30



Module 2

The Conservation of Momentum

Distance Learning

Alberta
EDUCATION

Physics 30

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The Conservation of Momentum

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EDUCATION

Distance Learning

This document is intended for	
Students	✓
Teachers (Physics 30)	✓
Administrators	
Parents	
General Public	
Other	

Physics 30
Student Module
Module 2
The Conservation of Momentum
Alberta Distance Learning Centre
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Cover Photo: PHOTO SEARCH LTD.

Welcome to Module 2!

We hope you'll enjoy your study of *The Conservation of Momentum*.

To make your learning a bit easier, watch the referenced videocassettes whenever you see this icon.

You also have the option of viewing laser videodisc clips when you see the bar codes like this one.

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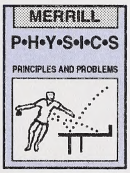
When you see this icon, study the appropriate pages in your textbook.



Good Luck!

Course Overview

This course contains nine modules. The first two modules develop the conservation laws of energy and momentum. The conservation of energy is at the heart of the entire course. Modules 3 through 9 build one upon the other and incorporate the main ideas from the preceding modules.

The module you are working in is highlighted in a darker colour.

PHYSICS 30

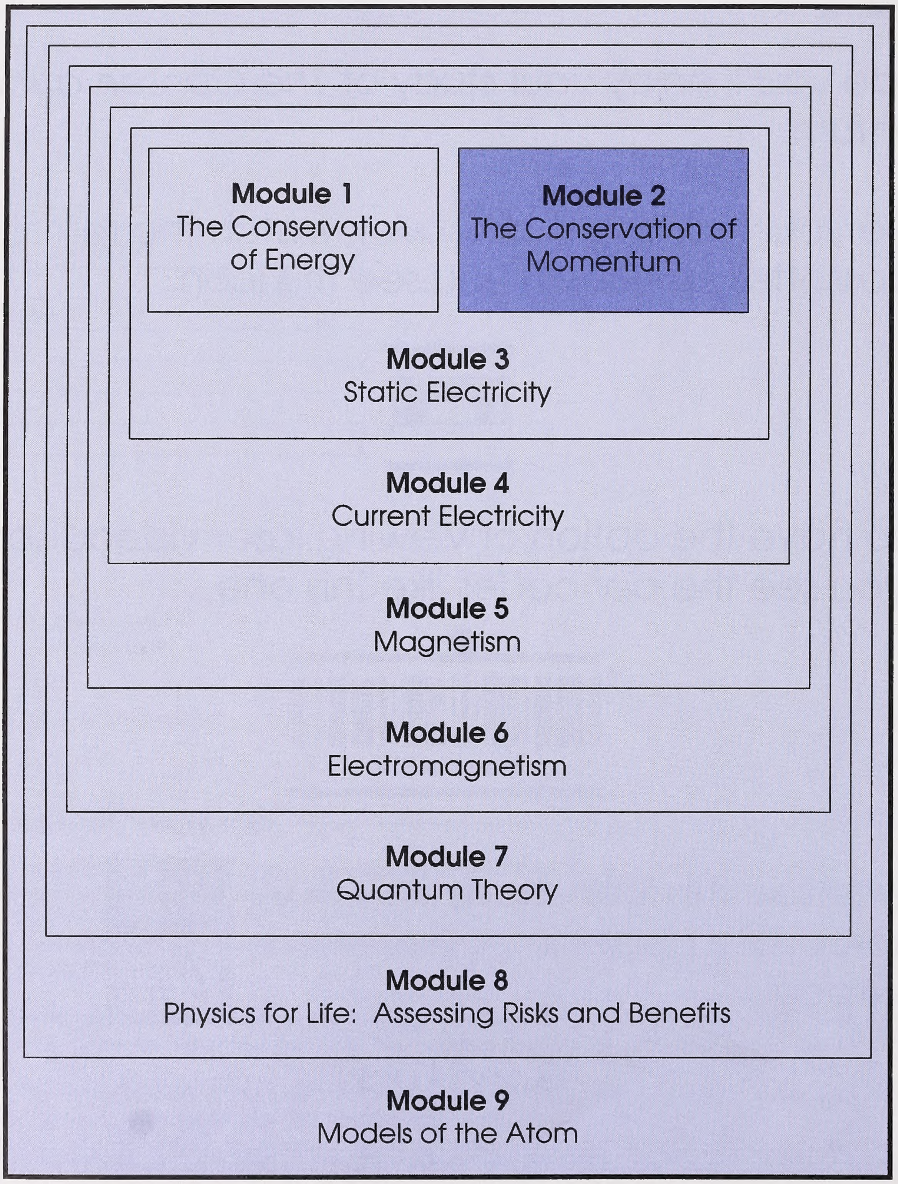


Table of Contents

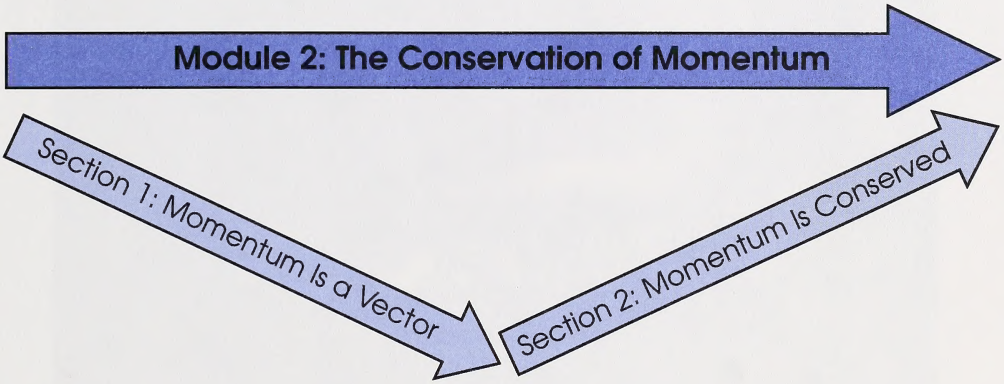
OVERVIEW

Your whole body seems like it is on sensory overload. Your heart is pounding, the knots in your stomach twist even tighter, and your mind is racing with dozens of conflicting thoughts. The bungee jump looked so harmless from below, but now that you are on the tower receiving final instructions, it seems much higher. The only thing sustaining you is that you will be able to tell your friends that you did it.

"Ready?" The instructor's voice seems to come from another world. All you can do is nod and close your eyes as you lean over the edge to begin your descent.

Is bungee jumping something that appeals to you? Would you enjoy the thrill of a bungee jump, or would you prefer to be a spectator?

People who bungee jump are actually putting their trust in a sophisticated cord system that is based on the physics of momentum. In this module you will explore how the concept of momentum can be applied to this situation and others.



Evaluation

Your mark in this module will be determined by your work in the Assignment Booklet. You must complete all assignments. In this module you are expected to complete two section assignments. The mark distribution is as follows:

Section 1 Assignment	42 marks
Section 2 Assignment	58 marks
TOTAL	**100 marks**

Section

1 Momentum Is a Vector

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Imagine that you are a famous billiards champion. You lean over the table and take careful aim at the cue ball. In your mind you imagine the shot and the resulting collision between the balls.

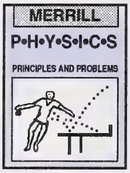
Since direction plays a key role in the success of your shot, this situation actually requires a vector analysis to occur in your mind. You must consider the forces, velocities, and outcomes of collisions if your shot is to be successful. Momentum is another vector quantity that you need to consider in order to make this shot.

In this section you will reinterpret the vector material that you first studied in Physics 20. You will also combine the concepts of force and motion in an introductory look at momentum. Momentum and impulse provide the link between Newton's laws and the study of collisions.

Activity 1: Reviewing Vectors

Have you ever arrived fifteen minutes late for a movie? Even though you eventually pick up the plot and manage to enjoy yourself, you know that it is still not quite the same experience that you could have enjoyed had you been there from the beginning. If the movie has a complex plot, the effects of missing the beginning can be worse. You may never really feel like you are able to catch the whole meaning of the film.

This first activity of Module 2 is much like the beginning of a movie with an intriguing plot. If you miss the main ideas here, you may feel out of touch for the rest of the course. Although the ideas are meant to be a review of Physics 20, you will have to pay close attention to the details of how to properly communicate vector solutions. It is important to learn how to communicate your solutions in a way that will help you be successful in the whole course, including the diploma exam. So, as you review vectors, be sure to check your answers closely with the solutions in the Appendix.



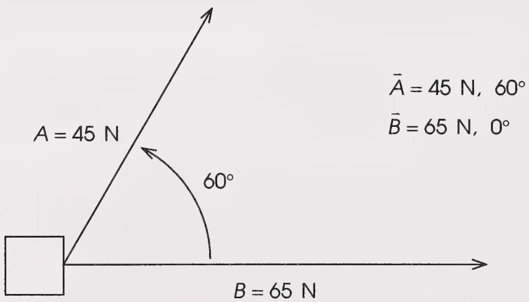
Begin your review of vectors by reading the introduction and the section called Vector Addition in One Dimension on page 110 of your textbook. Answer the following questions to help highlight the key points.

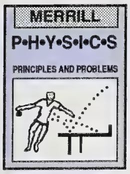
1. What is a *resultant*? How is a resultant drawn?

2. Does order matter when you add vectors together?

Vectors can also be added in two dimensions. Read from the bottom of page 110 to the end of the first paragraph on page 111 of your textbook. Be sure to take special note of Figure 6-2.

The angle between two vectors does not always have to be a right angle. For example, the two vectors could be arranged as shown in the following diagram.





How would you add these two vectors together? You can find the answer to this question by reading the middle paragraph on page 111 and by closely examining the right half of Figure 6-3.

3. Use a ruler and a protractor to show that you will get the same answer if you start by drawing $\vec{A}$ first.

The fact that order doesn't matter when you add two-dimensional vectors also applies when three or more vectors are added together. This is explained and illustrated on the bottom of page 111 of your textbook. Read this section and then read the section called Independence of Vector Quantities on page 112. When you are done reading, do the following questions.

4. Do Practice Problems 1. b. and 5 on page 112 of your textbook. Solve each problem using a ruler and a protractor.

Check your answers by turning to the Appendix, Section 1: Activity 1.

Up to this point the approach that you have been using has largely been graphical – solutions could be obtained by drawing the vectors with a ruler and a protractor. However, often a more precise answer is required, so an alternative mathematical approach is used in these cases.

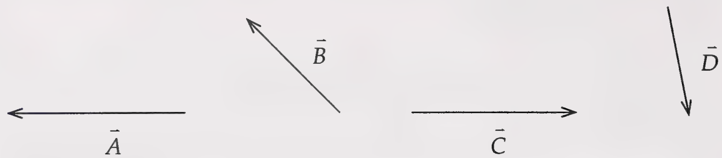
Your textbook begins to use a system of vector notation that is inconsistent with what you are expected to use on the diploma exam. The following guidelines explain the proper use of vector notation in Physics 30:

- You should be able to add and subtract vector quantities from each other. You should also be able to multiply and divide vector quantities by constants or by scalar quantities.

- You are **not** expected to multiply and divide vector quantities by other vector quantities. The only time that vectors should be multiplied or divided occurs when both vectors are colinear, in which case they can both be treated like scalars.

You have already learned how to add and subtract vectors in Physics 20. The multiplication of vectors is a topic that most students study in college or university, so you don't need to worry about it for now.

Use the following vector diagrams to answer question 5.



5. Copy the following headings into your notebook. Be careful to leave enough space under each heading to record your answers. Complete the chart by explaining why each operation is or is not permitted in Physics 30.

Proper Use of Vector Notation in Physics 30	
Operation	**Is the vector notation consistent with what is expected in Physics 30?**
$3\vec{A}$	
$\vec{A}+\vec{B}$	
$\vec{A}\times\vec{C}$	
$-359\vec{D}$	
$\vec{C}-\vec{D}$	
$\dfrac{\vec{B}}{\vec{C}}$	
$m\vec{B}+n\vec{C}$	

Check your answers by turning to the Appendix, Section 1: Activity 1.

Now that you know how to properly use vector notation, you can return to your textbook. Unfortunately, the first printing of the Canadian edition of the textbook is not consistent with the accepted use of vector notation for Physics 30. This means that if you are using this version of the text, you will have to be very careful when studying the Example Problems and the answers on pages 656 to 687. Fortunately, you have the examples and the answers in the Appendix of each student module booklet to work from. Each time the textbook presents vectors in an inconsistent way, it will be brought to your attention.

Carefully read pages 113 and 115 of your textbook to learn how trigonometry can be applied to the solution of vector problems.

6. What notation that is used in the top paragraph of page 115 is inconsistent with the Physics 30 notation? Rewrite it properly.

Although the Example Problem on page 115 in the textbook shows a good overall approach, the notation is inconsistent with the expectations of Physics 30. Here's how this problem would be solved using the recommended notation. Hints explaining the strategies have been put in boxes.

Example

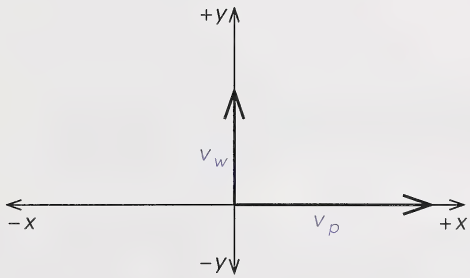
$\vec{v}_p = 90.0\,\text{km/h},\ 0°$

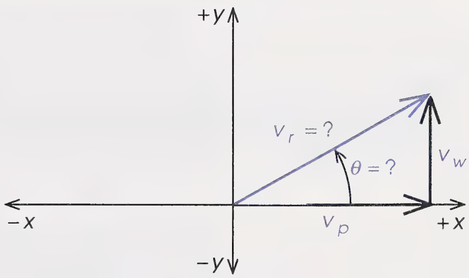
$\vec{v}_w = 50.0\ \text{km/h},\ 90°$

$\vec{v}_r = ?$

Begin the problem by listing the data.

Draw the vectors on an axis, showing the angle (θ) measured counterclockwise from the x-axis.





Step 1: Find the magnitude of the resultant.

$$C^2 = A^2 + B^2$$

$$(v_r)^2 = (v_p)^2 + (v_w)^2$$

$$= (90.0\ \text{km/h})^2 + (50.0\ \text{km/h})^2$$

$$= 10\,600\ \text{km}^2/\text{h}^2$$

$$v_r = 103\ \text{km/h}$$

These calculations involve squaring and dividing, so drop the vector notation.

Step 2: Find the angular dimension of the resultant.

$$\tan\theta = \frac{v_w}{v_p}$$

$$= \frac{50\ \text{km/h}}{90\ \text{km/h}}$$

$$= 0.5\overline{5}$$

$$\theta = 29.1°$$

Express the final answer in full vector form, stating magnitude and direction.

$$\vec{v}_r = 103\ \text{km/h},\ 29.1°$$

Drawing a clearly labelled vector diagram early in a solution is very important. Use the approach described in the previous example to solve the following problems.

7. Do Practice Problems 7 and 10 on pages 115 and 116 of your textbook.

Just as two vectors can be replaced with one vector, it is possible to represent one vector with two vectors. Carefully read from page 116 through to the end of the Example Problem on page 118 in the textbook.

8. The algebra shown in the paragraph under Figure 6-9 on page 117 in the textbook does not use the recommended Physics 30 notation. Redo this work to show the appropriate notation.

9. Redo the Example Problem on page 118 in the textbook using the appropriate notation.

10. Do Practice Problems 11 and 12 on page 118 of your textbook.

The technique of resolving vectors into components can be used to mathematically solve for the resultant of two vectors which are not at right angles to each other. This is shown on page 119 of your textbook. Read this page carefully, paying special attention to Figure 6-12.

11. Redo the Example Problem on page 119 of your textbook by using the notation that is recommended for Physics 30.

12. Do Problems 17, 19, 23, and 26 on page 130 of your textbook. It is very important to begin each solution with a vector diagram.

Check your answers by turning to the Appendix, Section 1: Activity 1.

momentum – the product of the mass and velocity of an object ($\vec{p} = m\vec{v}$)

impulse – the product of the force acting on an object and the time interval over which it acts ($\vec{F}\Delta t$)

Activity 2: Introducing Momentum

Look at the photograph on the cover of this module. What physics terms come to mind to describe the motion of the young woman bungee jumping?

You probably thought of terms like *speed*, *velocity*, *force*, *acceleration*, and *energy*. Two additional terms that could be used are **momentum** and **impulse**. It's interesting to know that these are the terms that Isaac Newton originally used to describe his three laws of motion. To find out more about these new terms, carefully read from the top of page 176 to the bottom of page 178 in your textbook.

1. How are the concepts of momentum and impulse related?

2. A soccer ball has a mass of 0.840 kg and is moving at 3.4 m/s, east.

   a. How much momentum does the ball possess?

   b. Arnold kicks the ball with an average force of 20.0 N, east. This force is exerted over a short time interval of 0.220 s. What impulse is given to the ball?

   c. With what velocity does the ball leave Arnold's foot?

Check your answers by turning to the Appendix, Section 1: Activity 2.

Isaac Newton developed the three fundamental laws of motion. The first and second laws deal directly with the motion of an object.

3. Copy the following headings into your notebook. Be careful to leave enough space under each heading to accomodate your answers. Complete the chart by describing the relationship between Newton's first and second laws of motion and momentum.

Newton's Law	Main Idea	How it Relates to Momentum
First		
Second		

If you examine Newton's second law closely, you will see that $\vec{F} = m\vec{a}$, or $\vec{F} = m\left(\frac{\Delta \vec{v}}{\Delta t}\right)$, can be stated as $\text{force} = \text{mass} \times \text{rate of change of velocity}$. However, if you take this one step further, you will get $\vec{F} = \left(\frac{m\Delta \vec{v}}{\Delta t}\right)$, or $\vec{F} = \left(\frac{\Delta \vec{p}}{\Delta t}\right)$. This equation could be restated as $\text{force} = \text{rate of change of momentum}$.

4. What is an advantage of using $\vec{F} = \frac{\Delta \vec{p}}{\Delta t}$ instead of $\vec{F} = m\vec{a}$? Explain your answer by referring to the launch of a space shuttle.

As stated and shown on page 177 of your textbook, the impulse given to an object is equal to the change in its momentum.

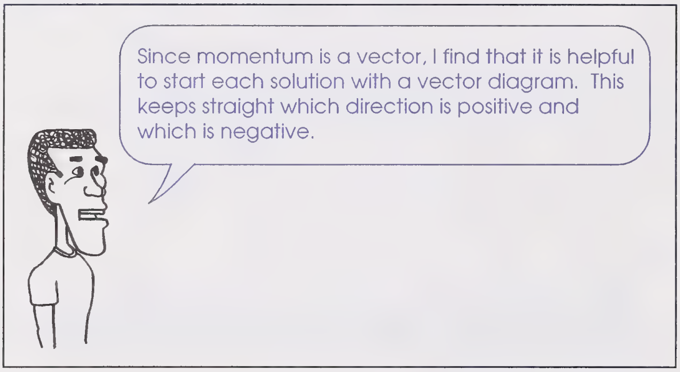
$$\vec{F}\Delta t = \Delta \vec{p}$$

5. a. Show the complete derivation of the impulse equation, beginning with the popular formula for Newton's second law, $\vec{F} = m\vec{a}$.

MERRILL PHYSICS PRINCIPLES AND PROBLEMS

b. If you read through the solution to the Example Problem on page 178 of your textbook, you will find that part b uses the impulse-momentum equation to determine the force exerted by the bat. Provide an alternative solution to part b. Which method of solution is preferable?

Check your answers by turning to the Appendix, Section 1: Activity 2.



Use Mike's suggestion as you complete the following questions.

MERRILL PHYSICS PRINCIPLES AND PROBLEMS

6. Do Practice Problems 2 and 3 on page 179 of your textbook.

7. Do Practice Problem 4 on page 179 of your textbook.

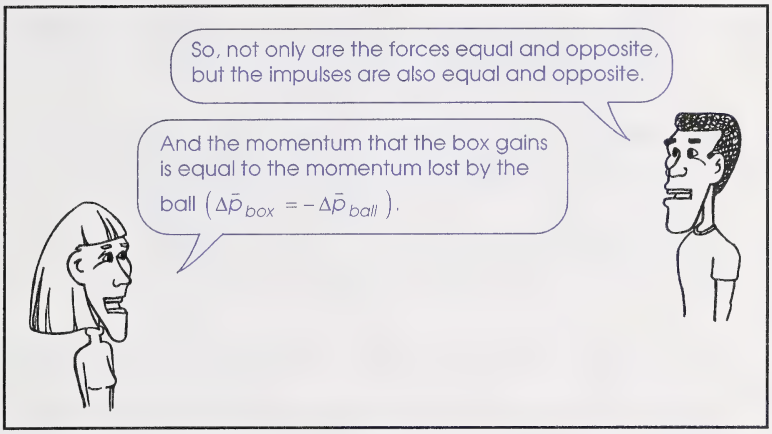
Check your answers by turning to page 669 in your textbook.

Science Skills

- ☐ A. Initiating
- ☐ B. Collecting
- ☐ C. Organizing
- ☑ D. Analysing
- ☐ E. Synthesizing
- ☐ F. Evaluating

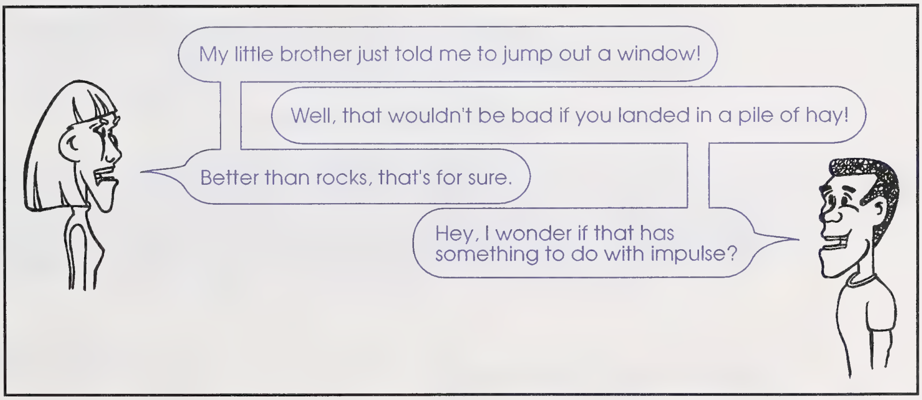
8. a. Figure 9-1 on page 176 of your textbook is a graph showing the force exerted by a ball on a box. This was the graph that you referred to in the previous question. Consider the reaction force exerted by the box on the ball. How would the graph of this force differ from Figure 9-1? How would the impulse differ from the impulse in question 7?

b. Consider your answer to question 8. a. What could be said about the change in the momentum of the ball as compared to the change in the momentum of the box?



Check your answers by turning to the Appendix, Section 1: Activity 2.

Sometimes the applications of momentum come from the strangest places. Nicole found that an argument with her younger brother actually helped identify an application of momentum.

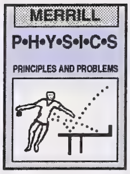


Imagine these two scenarios. First imagine Nicole jumping out a window into a pile of hay. Then imagine the same jump, but this time onto a pile of rocks. Use this example to answer questions 9 and 10.

9. Compare the change in momentum for each situation. What does this imply about the impulse given to Nicole in each jump?

10. Suppose that Nicole is moving at a speed of 3.00 m/s when she hits each pile. It takes her 1.130 s to come to rest in the hay, but in the pile of rocks it takes her only 0.245 s. Nicole's mass is 50.0 kg. Use calculations to show why it hurts more to jump into the rocks.

11. Explain some of the essential characteristics of a well-designed bungee cord. Your answer should include the terms *momentum*, *impulse*, *force*, and *energy*.



The most practical example of a system designed to change momentum with a larger time interval and a smaller force is explained on pages 175, 176, and 177 of your textbook. Reread these pages and then answer the following question.

12. Do Applying Concepts question 2 on page 192 of your textbook.

The following questions are an appropriate way to end this section because they get to the heart of the idea that momentum is a vector.

13. Do Concept Review question 1.1 on page 180 of your textbook.

14. A billiard ball with a mass of 434 g is moving with a velocity of 5.8 m/s at an angle of 218°.

    a. Calculate the magnitude of the momentum in units of kg • m/s.

    b. Sketch the momentum vector on an *x-y* axis to show its direction. Show the angle of 218°.

    c. Draw the *x*- and *y*-components of the momentum on the sketch that you drew to answer question 14. b.

    d. Calculate the *x*- and *y*-components of the momentum.

Check your answers by turning to the Appendix, Section 1: Activity 2.

Follow-up Activities

If you had difficulties understanding the concepts in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts, it is recommended that you do the Enrichment.

Extra Help

1. Classify each of the following terms as a vector or a scalar.

temperature	speed	age	impulse
momentum	force	acceleration	time
mass	distance	displacement	velocity

2. List the two equations needed to resolve a momentum vector, $\vec{p}$, into its components, if you know both the magnitude and direction of the original momentum.

In order to perform vector addition, the following steps must be performed in the proper order:

- Resolve each vector into components.
- Add x-components together and add y-components together.
- Use the Pythagorean theorem to determine the magnitude of the resultant vector.
- Use the tangent ratio to determine the direction of the resultant vector.

3. Do Practice Problem 16 on page 120 of your textbook.

4. Copy the following headings into your notebook. Be careful to leave enough space under each heading to record your answers. Complete the chart by writing the symbol, equation, and units for each of the quantities.

Quantity	Symbol	Equation	Units
Force			
Impulse			
Momentum			

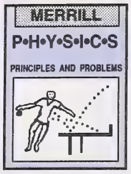
Check your answers by turning to the Appendix, Section 1: Extra Help.

Enrichment

Do **one** of the following Enrichment activities.

1. Perform the Pocket Lab on page 121 of your textbook. You will need help from two strong friends. Draw a vector diagram and write an explanation for your observations.

2. If you have access to the booklet *Supplemental Lessons*, which accompanies the Teacher Resource Package for your textbook, read through the Supplemental Lesson 1 section titled The Moment of Inertia on page 7. Answer Practice Exercises 14, 15, and 16 on page 8. This leads to the section titled Angular Momentum on pages 11 to 13 of the booklet. After reading this material you can answer Practice Exercises 3, 4, and 5 on pages 13 and 14 of this booklet.



3. Read Physics and Technology on page 187 of your textbook and answer the question at the end of the reading.

Check your answers by turning to the Appendix, Section 1: Enrichment.

Conclusion

In this section you reviewed the vector concepts that are an important part of Physics 30. You were also introduced to momentum as another way of measuring motion. Force was related to motion by analysing the impulse provided.

In the next section you will explore momentum in greater detail. You will see that conservation of momentum is a fundamental concept that governs all interactions.

Assignment Booklet

ASSIGNMENT

Turn to your Assignment Booklet and do the assignment for Section 1.

Section

2 Momentum Is Conserved

WESTFILE INC.

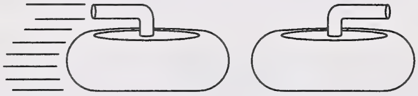
The specialists who examine the evidence from collisions and other automobile accidents use physics in their investigations. Skid marks can be used to estimate the speed of a vehicle prior to braking by using the coefficient of friction. Two-dimensional collisions can be analysed in terms of the interaction of the momentum vectors.

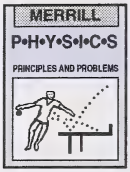
In this section you will find a way to predict whether momentum will be conserved in a collision between any two objects. Next you will apply the law of conservation of momentum by analysing collision data and by solving a variety of problems. All of these ideas will be used to explain the outcomes of collisions in one and two dimensions.

Finally, you will combine the conservation laws of momentum and energy by examining the nature of those laws in different types of collisions.

Activity 1: Isolated Systems

Many sports involve collisions. In hockey and football it is usually the athlete's bodies that are in collisions. In curling or billiards the rocks or balls are having numerous collisions.





In each of these collisions, the initial momentum of each body plays an important role in the outcome.

To help you understand the relationship between momentum and collisions, read pages 180, 181, and 183 in your textbook. As you read, look for ideas that are similar to those that you learned in Module 1.

closed system – a system in which the total mass remains constant

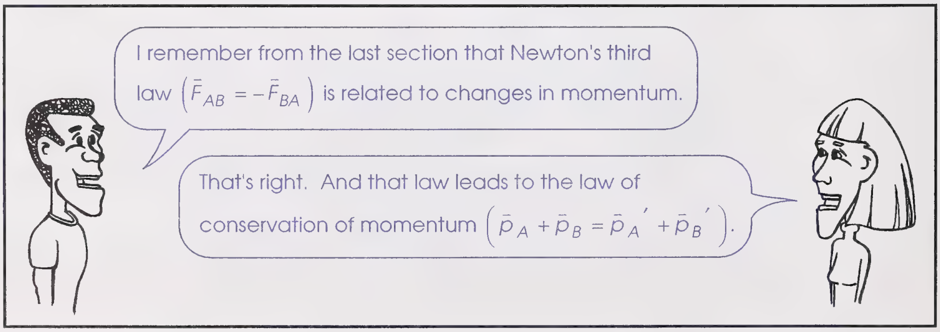
isolated system – a system in which no net force is exerted on it

1. a. Use the definition of a **closed**, **isolated system** to describe how the air table used in Module 1 can be classified as a closed, isolated system.

   b. Answer Reviewing Concepts question 3 on page 192 of the textbook.

2. Suppose the baseball player described on page 180 of the textbook had no spikes on his shoes and was standing on ice. How would this change the outcome of the collision between the bat and the ball?

3. Use the following discussion between Mike and Nicole to derive the law of conservation of momentum using Newton's third law.



4. Write the algebraic expression for each of the following statements. The system you are describing is a closed, isolated system with two objects labelled A and B.

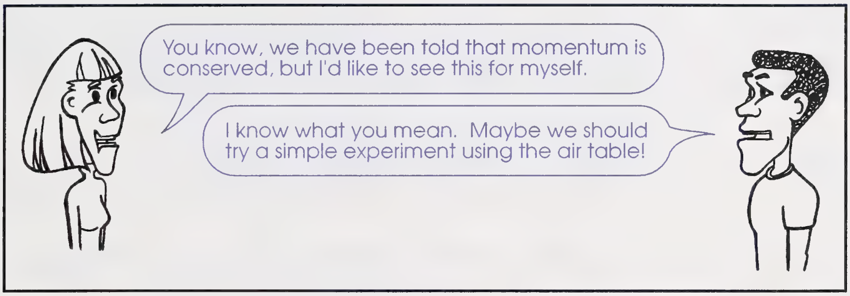
   a. The net change in momentum is zero.

b. The total momentum does not change during a collision.

Check your answers by turning to the Appendix, Section 2: Activity 1.

Summation notation is useful for dealing with the conservation of energy. The answer for question 4. b. could easily be written as $\sum \vec{p}_{before} = \sum \vec{p}_{after}$. This is the method that most simply expresses the law of conservation of momentum.

Activity 2: Momentum in a Straight Line



Investigation: Conservation of Momentum in One Dimension

Science Skills

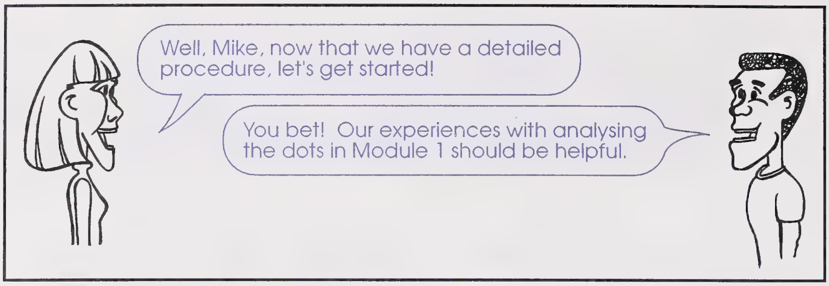
- ☑ A. Initiating
- ☐ B. Collecting
- ☐ C. Organizing
- ☐ D. Analysing
- ☐ E. Synthesizing
- ☐ F. Evaluating

1. Help Mike and Nicole design an experiment which will test the law of conservation of momentum. To make the experiment simple, keep the following points in mind.

   - Assume that Mike and Nicole have access to an air table.
   - Use the simplest type of collision. In this type of collision (linear), the pucks collide in one direction only and the two air pucks stick together after the impact.

   Include in your design a list of materials, a detailed procedure, and indicate potential problems that may occur. Also note any safety precautions that should be observed while using the high-voltage spark timer.

Check your answers by turning to the Appendix, Section 2: Activity 2.



PATHWAYS

If you have access to an air table in a supervised science lab, do Part A.
If you do not have access to an air table, do Part B.

Part A

Caution

Science Skills

- ☐ A. Initiating
- ☑ B. Collecting
- ☑ C. Organizing
- ☑ D. Analysing
- ☐ E. Synthesizing
- ☐ F. Evaluating

Perform the investigation outlined in your answer to question 1. Read through the complete procedure before beginning and exercise caution with the high-voltage tabletop.

2. Take all appropriate measurements and prepare a data chart that is suitable for analysing the momentum before and after the collision. Pay particular attention to the time interval setting on the spark timer. Show all equations and calculations.

3. Analyse the data to determine if momentum is conserved. If you consider the initial momentum to be the actual value and the final momentum to be the experimental value, percent error may be calculated.

Check your answers by turning to the Appendix, Section 2: Activity 2.

End of Part A

Part B

Science Skills

- ☐ A. Initiating
- ☑ B. Collecting
- ☑ C. Organizing
- ☑ D. Analysing
- ☐ E. Synthesizing
- ☐ F. Evaluating

In the Appendix you will find a pull-out page showing the path of a single air puck before collision and the two paths of the air pucks after they collide and stick together. Remove this sheet from the Appendix and use it as your air-table data. Be sure to check the title of the investigation at the top of the pull-out page, as there is more than one of these pages. The title on the pull-out page should match the title of this investigation.

Carefully read through the answer to question 1 in the Appendix before proceeding. This outlines the procedure and provides you with some idea of where the dots come from.

4. Take all appropriate measurements and prepare a data chart that is suitable for analysing the momentum before and after the collision. It is important that you have the following information:

   mass of each puck = 550 g
   spark timer interval = 20 ms

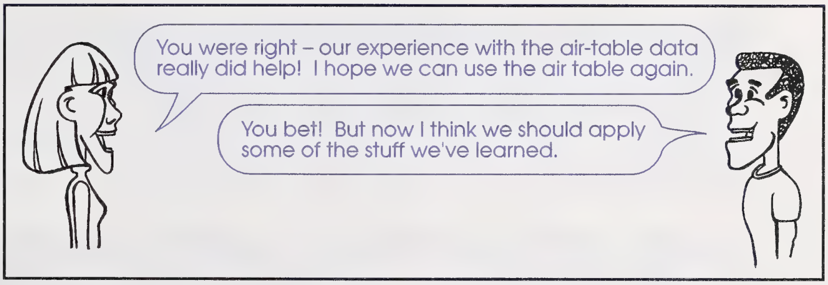
   Show all equations and calculations.

5. Use the initial momentum as the actual value and the final momentum as the experimental value to determine the percent error. Is it possible to conclude that momentum is conserved?

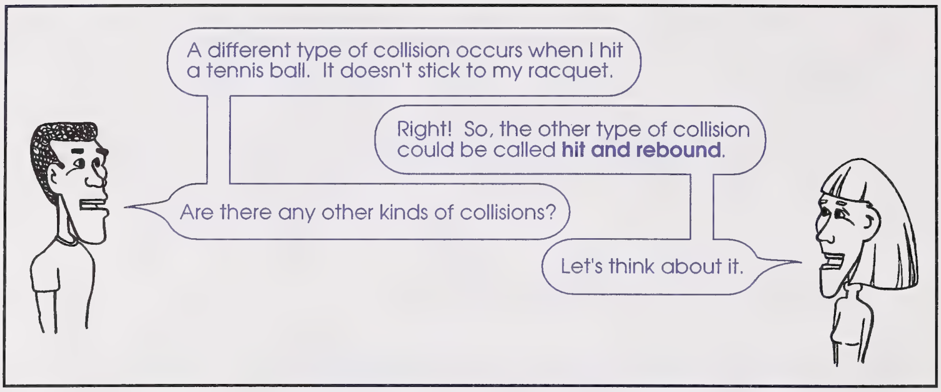
Check your answers by turning to the Appendix, Section 2: Activity 2.

End of Part B

Activity 3: Applying the Law of Conservation of Momentum



The investigation that you just completed involved a very simple collision. The air-table interaction involved two pucks which stuck together on impact. Not all collisions have that same result. You will take a close look at two-dimensional collisions in Activity 4. In this activity you will examine the different types of one-dimensional collisions.



1. a. Imagine Mike and Nicole standing face to face on skates. They are both at rest. What is their initial momentum?

   b. Now suppose they push away from each other. Will momentum be conserved? How do you know? What is the sum of their final **momenta**?

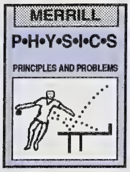
momenta – the plural form of momentum

   c. Assuming that Mike is more massive than Nicole, whose resulting speed will be greater? Start your solution with a diagram that clearly shows the variables and the sign convention.

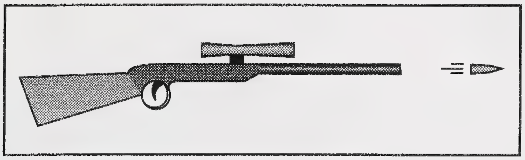
2. Copy the following headings into your notebook. Be careful to leave enough space under each heading to record your answers. Complete the chart by adding the other two general types of collisions and one distinguishing feature of each. As well, write a simpler version of the algebraic expression $m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}_A{}' + m_B\vec{v}_B{}'$ for the other two collision types.

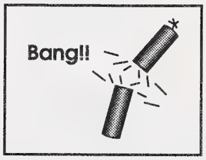
Type of Collision	Distinguishing Feature	Equation
Hit-and-rebound	After collision, the two (or more) bodies are still separate.	$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}_A{}' + m_B\vec{v}_B{}'$

Check your answers by turning to the Appendix, Section 2: Activity 3.



One type of collision, which could be classified as **explosion**, is easier to understand if you read the section called Internal and External Forces on pages 186 to 188 of your textbook.



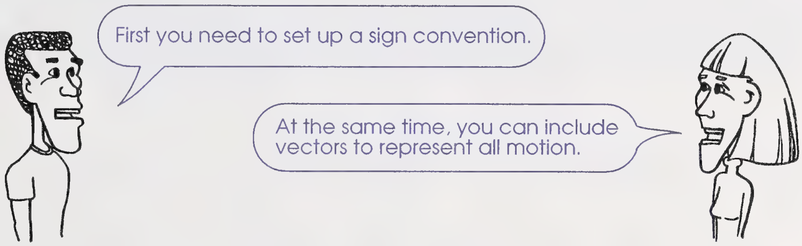


Take a close look at the solutions to the Example Problems on pages 185 and 188 of the textbook.

Pay very close attention to the methods and notations used by Mike and Nicole as they attempt the following sample problem.

Example

An air puck, A, with a mass of 550 g is travelling east at 38 cm/s when it collides with a second puck, B, with a mass of 715 g that is travelling west at 32 cm/s. Velcro strips cause the pucks to stick together. Assuming that there is no rotation by either puck, what is the common final velocity of the two pucks?

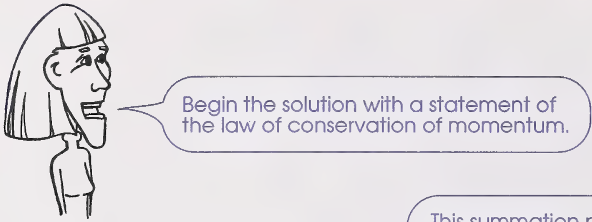


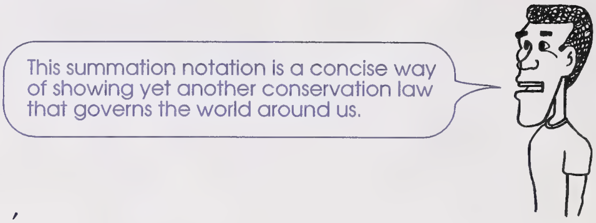
$\vec{v}_A = +38$ cm/s

$\vec{v}_B = -32$ cm/s

$m_A = 0.550$ kg

$m_B = 0.715$ kg

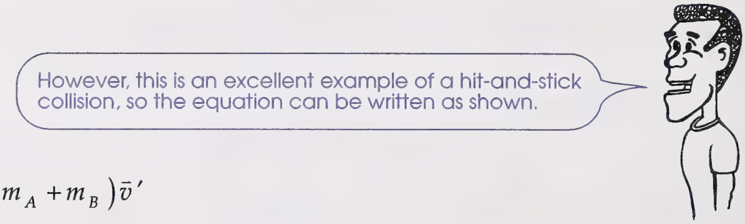




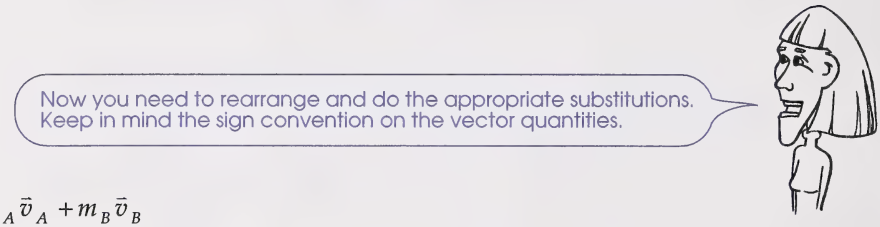
$$\sum \vec{p}_{before} = \sum \vec{p}_{after}$$

$$\vec{p}_A + \vec{p}_B = \vec{p}_A{}' + \vec{p}_B{}'$$

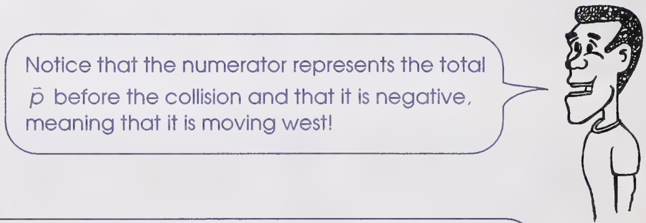
$$m_A \vec{v}_A + m_B \vec{v}_B = m_A \vec{v}_A{}' + m_B \vec{v}_B{}'$$

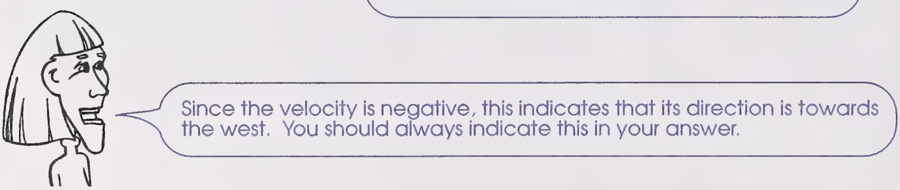


$$m_A \vec{v}_A + m_B \vec{v}_B = (m_A + m_B)\vec{v}\,'$$



$$\begin{aligned}
\vec{v}\,' &= \frac{m_A \vec{v}_A + m_B \vec{v}_B}{(m_A + m_B)} \\
&= \frac{(0.550\ \text{kg})(+38\ \text{cm/s}) + (0.715\ \text{kg})(-32\ \text{cm/s})}{(0.550\ \text{kg} + 0.715\ \text{kg})} \\
&= \frac{-1.98\ \text{kg}\bullet\text{cm/s}}{1.265\ \text{kg}} \\
&= -1.6\ \text{cm/s}
\end{aligned}$$





The velocity is 1.6 cm/s, west.

Do each of the following problems from the textbook. It is helpful to sketch a vector diagram and list the given information for each question.

3. Do Practice Problems 5, 6, and 7 on page 185 of your textbook.

4. Do Practice Problem 8 on page 185 of your textbook.

Check your answers by turning to the Appendix, Section 2: Activity 3.

Did you notice the slight variations between the solutions in the textbook and those found in the Appendix? The additional diagrams and notations in the Appendix are meant to improve your understanding of the vectors involved in momentum. Now do the following questions.

5. Do Practice Problems 9, 10, and 11 on pages 188 and 189 of your textbook.

6. Do Practice Problem 12 on page 189 of your textbook.

Check your answers by turning to the Appendix, Section 2: Activity 3.

You will find that the following group of questions will make you stop and think about the nature of momentum and isolated systems.

7. Do Reviewing Concepts questions 7 and 10 on page 192 of your textbook.

8. Do Problems 22, 25, and 27 on pages 194 and 195 of your textbook.

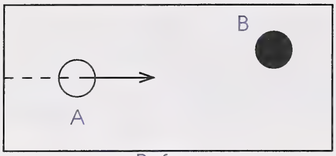
9. Do Applying Concepts questions 6 and 9 on pages 192 and 193 of your textbook.

Check your answers by turning to the Appendix, Section 2: Activity 3.

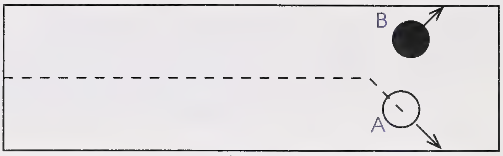
In this activity you have applied most of the main ideas of the conservation of momentum. However, there is another step for you to take. In the next activity you will begin working with momentum in two-dimensional collisions.

Activity 4: Collisions in Two Dimensions

Imagine a collision between two billiard balls or curling stones. You will probably imagine something like what is shown in the following diagrams.



Before



After

You will notice that this collision is not linear, like the ones you have dealt with in the previous two activities.

In the following investigation you will work with the air table and examine a two-dimensional collision.

Investigation: A Two-Dimensional Collision

Science Skills

- ☐ A. Initiating
- ☑ B. Collecting
- ☑ C. Organizing
- ☑ D. Analysing
- ☐ E. Synthesizing
- ☑ F. Evaluating

Purpose

In this investigation you will determine whether momentum is conserved in a collision in two dimensions.

PATHWAYS

If you have access to an air table, do Part A.
If you do not have access to an air table, do Part B.

Part A

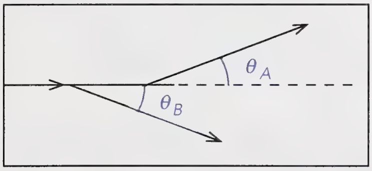
Materials

You will need the following materials for this investigation:

- air table with two air pucks
- puck launcher (if available)
- one piece of newsprint
- a balance

Procedure

- Read through each step carefully before doing the investigation.
- Measure the mass of each puck before or after performing the experiment.
- Make sure the air table is level by placing a puck in the centre of the table and turning on the air. When the table is level, the puck should be relatively motionless.
- Place one of the pucks in the centre of the newsprint on the air table. Try to have this puck situated so that the other puck will not hit it straight on, but rather at some angle.
- Turn on the air and spark timer. Set the spark timer at 20-ms intervals, or any other setting if 20 ms is not available.
- Launch or push the second puck towards the stationary puck.
- You may need to practise a few times in order to get both pucks to rebound at proper angles.



Both θ_A and θ_B should be greater than 20° for best results.

- Turn to the Observations section which follows Part B.

End of Part A

Part B

In the Appendix you will find a pull-out page that shows the path of a single air puck (Puck A) before the collision and two separate paths after the collision. Remove this page and use it as your source of data. Be sure to check the name of the investigation at the top of the page. The title on the pull-out page should match the title of this investigation.

Before you continue, read how the data was obtained in Part A.

Continue the investigation by turning to the Observations, Analysis, and Conclusion sections for this investigation. Keep the following measurements in mind as you proceed.

mass of puck A = mass of puck B = 550 g

spark timer interval = 20 ms

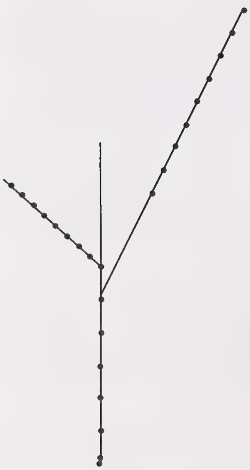
End of Part B

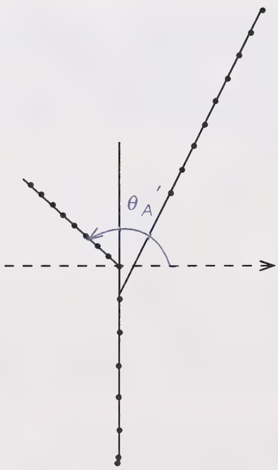
Observations

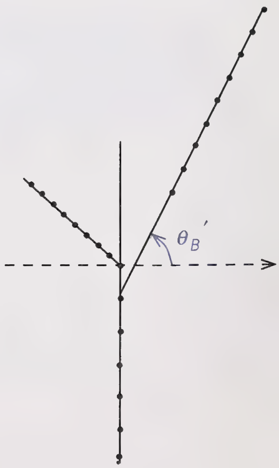
1. Copy the following headings into your notebook. Be careful to leave enough space under each heading to record your answers. Complete the chart by filling in the appropriate measurements and calculations.

	Before Collision		After Collision	
	Puck A	Puck B	Puck A	Puck B
Mass (kg)				
Distance (cm)				
Time (s)				
Magnitude of Velocity (cm/s)				
Momentum (kg • cm/s)				
Angle to Indicate Direction (°)	90	–		

In order to determine the angle, you will need to extend a line back to the original line along the dots as shown in the following three diagrams. Let the angle of puck A before the collision be along the positive y-axis, or 90°.

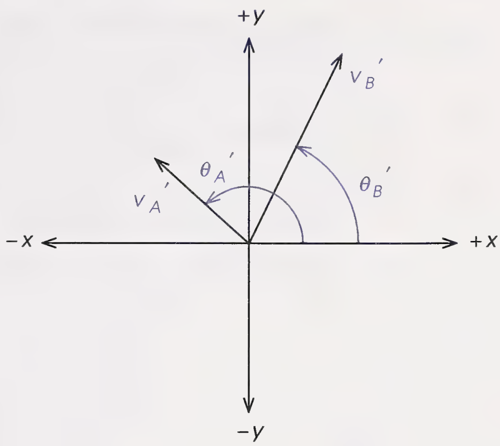






As shown in the previous three examples, it is not likely that all three lines will intersect, since the middles of the two pucks did not intersect.

As shown in the sketch to the right, it is important to measure all angles using the x-y coordinate system.



Analysis

2. Draw all three momentum vectors on an x-y coordinate system, with the initial momentum of puck A along the positive y-axis. Start each vector at the origin.

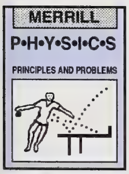
3. By resolving the x- and y-components, find the resultant of the two final momenta after the collision.

Conclusion

4. Compare the resultant found in question 3 to the initial momentum of puck A. Is momentum conserved in two dimensions?

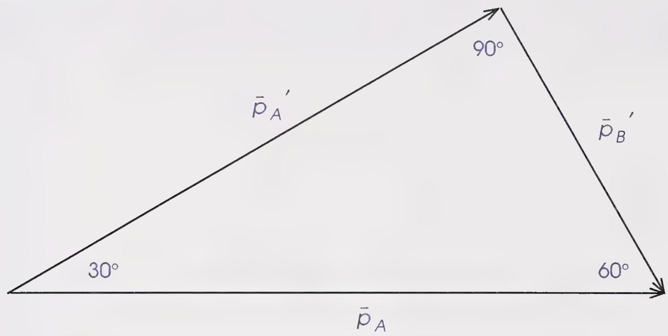
Check your answers by turning to the Appendix, Section 2: Activity 4.

Activity 5: Applications in Two Dimensions



Read through the section called Conservation of Momentum in Two Dimensions on pages 189 and 190 in your textbook. Then take a look at the Example Problem on pages 190 and 191. Although the textbook shows the use of vector notation with trigonometric functions, remember that you should avoid this practice. The division of vectors, as in $\frac{\vec{p}_A'}{\vec{p}'}$, is not acceptable notation for Physics 30. You should be dividing the magnitudes of these vectors only, such as $\frac{p_A'}{p'}$ or $\frac{|\vec{p}_A'|}{|\vec{p}'|}$.

The solution to this Example Problem also takes a slightly different approach than you took in the previous investigation. Rather than working with the components of the two final momenta, the Example Problem uses the **vector sum method**. Because of the law of conservation of momentum, you know that $\vec{p}_A + \vec{p}_B = \vec{p}_A{}' + \vec{p}_B{}'$, and since $\vec{p}_B = 0$, $\vec{p}_A = \vec{p}_A{}' + \vec{p}_B{}'$. The vector diagram for this sum will look exactly like the one at the top of page 190 in the textbook.



From this diagram, $p_A{}'$ and $p_B{}'$ can be found easily using trigonometry once p_A is calculated. This method is far easier than using components. In fact, the use of components would supply the correct solution, but only after solving a system of two equations.

1. The vector sum method is obviously very useful, but it has limitations. When is this convenient method not easily used?

So, if you can construct a vector sum diagram that includes a right angle, the vector sum method is available. The solutions in the Appendix for the problems that follow may or may not use this method.

As you work through some of these problems, keep the following points in mind:

- When resolving a vector $(\vec{a})$ at an angle (θ) into components, use the following scalar equations:

$$a_x = a(\cos\theta)$$
$$a_y = a(\sin\theta)$$

- If there is only one moving object before collision, arbitrarily place its vector along the positive x-axis (0°). This makes it easier to work with, as there is no initial y-component.

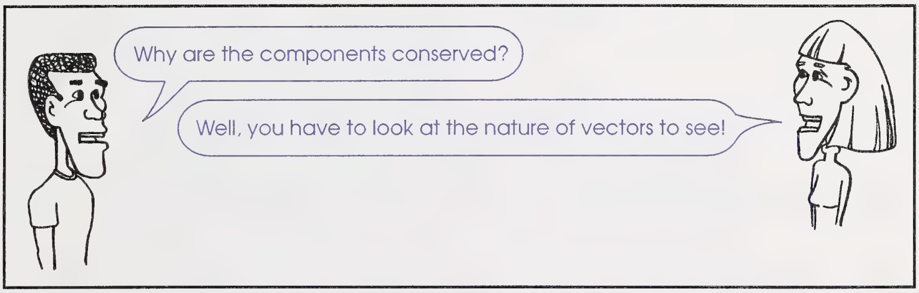
- Two-dimensional collisions will always have resulting momenta that follow two basic rules:

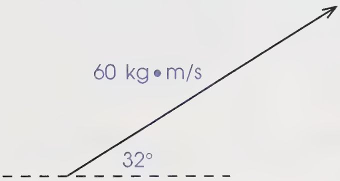
Rule 1: sum of x-components before collision = sum of x-components after collision

$$\sum p_x = \sum p_x'$$

Rule 2: sum of y-components before collision = sum of y-components after collision

$$\sum p_y = \sum p_y'$$





A single vector can represent both the total initial and total final momentum since momentum is conserved. Suppose the vector shown to the left represents the vector sum of individual momenta. It follows that the sum of the x-components and the sum of the y-components must be represented by the components of this vector.

$$\sum p_x = (60\ \text{kg}\bullet\text{m/s})(\cos\ 32°) = 51\ \text{kg}\bullet\text{m/s}$$

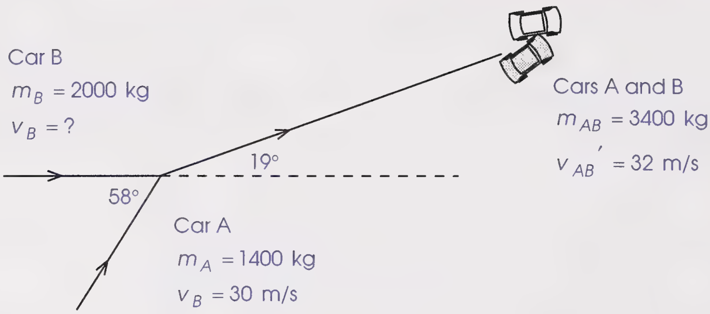
$$\sum p_y = (60\ \text{kg}\bullet\text{m/s})(\sin\ 32°) = 32\ \text{kg}\bullet\text{m/s}$$

MERRILL PHYSICS PRINCIPLES AND PROBLEMS

2. Do Practice Problems 13 and 14 on page 191 of your textbook. (Hint: Watch for vector sum diagrams containing a right angle.)

Check your answers by turning to pages 670 and 671 in your textbook.

3. A police officer at the scene of an accident can determine the speed of one car (car A) from the pattern of the skid marks. He can also determine the combined speed of the two cars by the distance they travelled together after the impact. The vector diagram of the collision and other pertinent information is provided for you.



Use this information to determine if car B was exceeding the posted speed limit of 110 km/h (31 m/s). (Hint: Since the vector sum diagram does not contain a right angle, you will need to use components. Review the hints at the beginning of this activity to help you solve this problem.)

4. On the final tear-out page in the Appendix, you will find results that could represent data from an air-table investigation. This time both pucks A and B are in motion and they have different masses. An *x-y* coordinate system has been introduced to make it easier to measure angles. Use the information found on the sheet and measurements that you make to determine the total initial momentum and the total final momentum. Is momentum conserved? (Hint: Once you have determined the actual momenta, you will need to resolve each of them into components before you proceed. From that point on, you will never use the actual values again.)

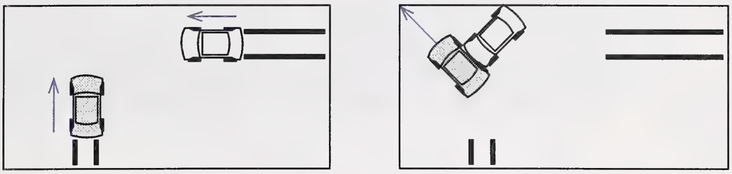
Check your answers by turning to the Appendix, Section 2: Activity 5.

Activity 6: Elastic and Inelastic Collisions

In this module and Module 1 you have analysed two extremely important conservation laws.

1. List the two laws of conservation which you have used in this module and Module 1.

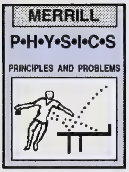
2. Imagine a collision between two automobiles, as shown in the following diagrams.



a. In the automobile accident shown in the previous diagrams, will momentum be conserved? Discuss the situation from the time of impact to the moment when the cars come to rest.

b. Is energy conserved? Describe the various forms of energy transfer that might take place.

Now imagine a collision between two billiard balls. This interaction is obviously quite different from the collision between the two cars.



3. Discuss the similarities and differences between a collision involving billiard balls and a collision involving cars.

Check your answers by turning to the Appendix, Section 2: Activity 6.

elastic collision – a collision in which kinetic energy is conserved

inelastic collision – a collision in which kinetic energy is not conserved

Read page 231 and the top third of page 233 in your textbook to learn about the meanings of the terms **elastic collision** and **inelastic collision**.

4. If you did not use calculations, what characteristics would you look for to differentiate between an elastic collision and an inelastic collision?

Your answers to the previous questions should indicate that a collision between billiard balls is highly elastic, while an auto accident is usually inelastic.

Almost all collisions will involve some amount of kinetic energy loss. It has been found that the closest thing to a perfectly elastic collision occurs on the molecular level between atomic and subatomic particles.

5. What will happen in a perfectly inelastic collision? Provide an example.

6. Answer Reviewing Concepts question 14 on page 236 of your textbook.

Check your answers by turning to the Appendix, Section 2: Activity 6.

To get a good sense of the differences between elastic and inelastic collisions, it is worthwhile to re-examine the data from Activities 2 and 4.

To assure some uniformity in results, use the following representative sample data for each activity. These results are shown in the following tables.

Activity 2 Investigation

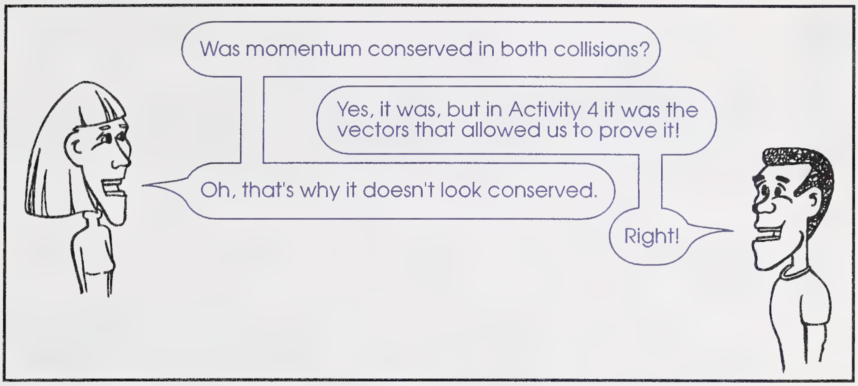
- one-dimensional, collide-and-stick collision

	Puck A Before Collision	Pucks A and B After Collision
Mass (kg)	0.550	1.100
Velocity (cm/s)	+ 65	+ 31
Momentum (kg • cm/s)	+ 36	+ 34

Activity 4 Investigation

- two-dimensional, collide-and-rebound collision

	Before Collision		After Collision	
	Puck A	Puck B	Puck A	Puck B
Mass (kg)	0.550	0.550	0.550	0.550
Magnitude of Velocity (cm/s)	83	0	38	63
Magnitude of Momentum (kg • cm/s)	46	0	21	35



7. a. Determine the total kinetic energy before and after the collision in the Activity 2 Investigation by using the data that was provided. Express your answers in joules.

   b. Calculate the percent of kinetic energy lost.

   c. Is this collision highly elastic? Explain.

8. a. Determine the total kinetic energy before and after the collision in the Activity 4 Investigation by using the data that was provided. Express your answers in joules.

   b. Calculate the percent of kinetic energy lost.

   c. Is this collision highly elastic? Explain

Check your answers by turning to the Appendix, Section 2: Activity 6.

Whenever two bodies stick together after a collision, there will be a greater amount of kinetic energy lost. This is due to the energy required to actually bind the two bodies together.

To see how these ideas can be applied to an automobile collision, read through the Example Problem on page 234 of your textbook. Note that vector notation is incorrectly applied to the kinetic energy equation. You should always write kinetic energy as a scalar $\left(E_k = \frac{1}{2}mv^2\right)$. The solution assumes the conservation of momentum. Without this assumption, the final velocity could not be found.

MERRILL P•H•Y•S•I•C•S PRINCIPLES AND PROBLEMS

9. Do Practice Problem 13 on page 234 of your textbook.

Now examine the Example Problem on page 233 of your textbook. Once again, errors are made by writing kinetic energy as a vector, and more assumptions are made in the solution. In this Example Problem, it is assumed that this is an elastic collision. This is necessary because there are two unknowns and two equations are needed.

10. Provide the missing steps in the derivations of $v' = \frac{m-M}{m+M}v$ and $V' = \frac{2\,mv}{m+M}$. Note that the vector notation has been dropped.

Check your answers by turning to the Appendix, Section 2: Activity 6.

It is important that you understand that these equations can only be used if one object is initially stationary.

11. Use the equation derived in the previous question to answer Practice Problem 15 on page 235 of your textbook.

Check your answers by turning to pages 673 and 674 in your textbook.

12. A 4.85-g bullet is fired at 214 m/s and becomes embedded in a 3.219-kg block resting on a frictionless surface. How much total kinetic energy will be lost immediately after the collision?

13. Magnetized air pucks are often used to reduce the friction of contact on an air table, and thereby increase the amount of kinetic energy conserved. Suppose a 0.510-kg air puck that is moving at 40 cm/s approaches a stationary magnetized 0.675-kg air puck. The collision is head-on, so the repulsive force causes the pucks to move along the same line. What is the velocity of each puck after this interaction?

Check your answers by turning to the Appendix, Section 2: Activity 6.

Follow-up Activities

If you had difficulties understanding the concepts in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts, it is recommended that you do the Enrichment.

Extra Help

1. There were three types of collisions discussed in this section. Specialized equations were developed for one-dimensional collisions between two objects.

$$m_A \vec{v}_A + m_B \vec{v}_B = m_A \vec{v}_A{}' + m_B \vec{v}_B{}'$$

$$m_A \vec{v}_A + m_B \vec{v}_B = (m_A + m_B)\vec{v}\,'$$

$$0 = m_A \vec{v}_A{}' + m_B \vec{v}_B{}'$$

   a. What type of collision is associated with each equation? Why is each equation written as it is?

   b. Which equation is the basic one which could be used in any problem involving two objects?

   c. You were asked to do Problem 22 on page 194 of your textbook in Activity 3. What type of collision is this? Which equation is best suited for this problem?

2. Do Problems 17 and 20 on page 194 of the textbook.

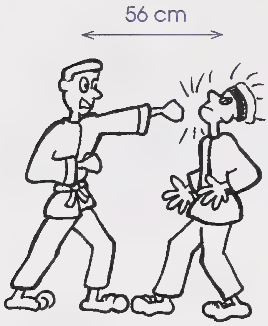
3. Use your answer to Problem 17 in question 2 to determine whether the collision is elastic or inelastic.

Check your answers by turning to the Appendix, Section 2: Extra Help.

Enrichment

Do **one** of the following Enrichment activities.

1. A laserdisc segment showing a hit in Tae Kwon Do is analysed by a group of students.



time to throw punch $= 3$ frames on the laserdisc

each frame on the disc $= \frac{1}{30}$ s

time of contact (less than 1 frame) estimate $= \frac{1}{2}$ frame

The fist and head move together during the time of contact. The following masses were given:

mass of fist and arm $(m_f) = 6.8$ kg

mass of head $(m_h) = 5.1$ kg

Determine the average force of the blow that was delivered.

2. Contact the provincial Department of Transportation and Utilities to locate a copy of the video called *A Crash Course on Belts and Bags* and its accompanying pamphlet, which provides a summary. Answer the six questions that are provided with the video.

3. Your head will hit the windshield if you are unrestrained in a head-on collision.

   a. Use the following sample data to determine the average force per unit area (pressure) exerted on your head.

      mass of head $= 5.0$ kg

      initial speed $= 10$ m/s (36 km/h)

      stopping time $= 2.0 \times 10^{-2}$ s

      contact area $= 6.0 \times 10^{-4}$ m^2

b. Now do the same question, except this time imagine that you are restrained by a seat belt. In this question your entire body is being stopped, not just your head.

$$\text{mass of body} = 70\ \text{kg}$$
$$\text{initial speed} = 10\ \text{m/s}$$
$$\text{stopping time} = 0.50\ \text{s}$$
$$\text{contact area} = 0.10\ \text{m}^2$$

c. Identify two ways that seatbelts help to reduce injuries.

4. In a perfectly elastic collision, the two conservation laws can be written algebraically as $m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1{}' + m_2\vec{v}_2{}'$ and $\frac{1}{2}m_1v_1{}^2 + \frac{1}{2}m_2v_2{}^2 = \frac{1}{2}m_1v_1{}'^2 + \frac{1}{2}m_2v_2{}'^2$.

Assume that both objects A and B are initially in motion and you are asked to find $v_1{}'$ and $v_2{}'$. Derive an equation that could be used to determine $v_1{}'$ and $v_2{}'$ in terms of m_1, m_2, v_1, and v_2. (Hint: Use systems of equations that are similar to the method used in the Example Problem on page 233 of your textbook. As well, be aware that the simple solution involves cancelling common factors whenever possible.)

Check your answers by turning to the Appendix, Section 2: Enrichment.

Conclusion

In this section you have demonstrated the law of conservation of momentum in both linear and two-dimensional collisions. The applications to problems showed how these ideas can be applied to many different situations.

Clearly, this conservation law is a powerful tool because it permits relatively easy analysis of complex events. When the conservation law of energy is combined with the conservation law of momentum, elastic and inelastic collisions can be analysed.

Assignment Booklet

ASSIGNMENT

Turn to your Assignment Booklet and do the assignment for Section 2.

MODULE SUMMARY

In this module you reviewed the vector concepts which later became an important tool for working with momentum. Vectors play an extremely important role in the following modules.

The major topic in this module was momentum. You began by looking at momentum as it relates to Newton's laws of motion. Later in the module the law of conservation of momentum was used to analyse experimental results and solve problems. Finally elastic and inelastic collisions were compared.

The world around you is filled with interacting objects. Hopefully this module has helped you look at these interactions in a new way.

Appendix

Glossary

Suggested Answers

Pull-out Pages

Glossary

closed system: a system in which the total mass remains constant

elastic collision: a collision in which kinetic energy is conserved

impulse: the product of the force acting on an object and the time interval over which it acts $\left(\vec{F}\Delta t\right)$

inelastic collision: a collision in which kinetic energy is not conserved

isolated system: a system in which no net force is exerted on it

momentum: the product of the mass and velocity of an object $\left(\vec{p} = m\vec{v}\right)$

resultant: the single vector that could represent the sum of several vectors

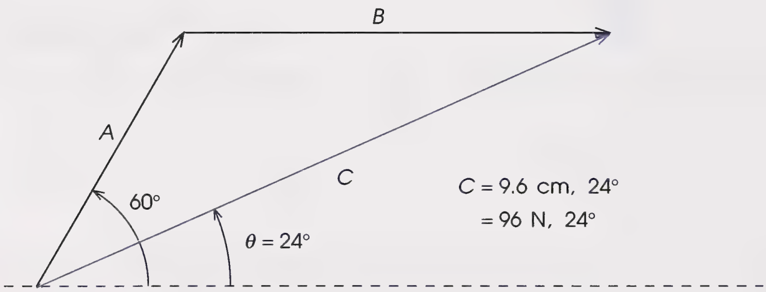
Suggested Answers

Section 1: Activity 1

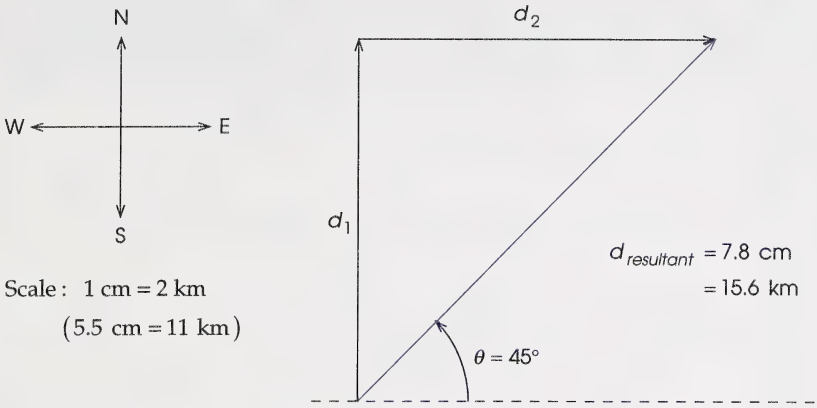
1. A resultant is a single vector that represents the sum of two or more other vectors. The resultant is shown graphically by an arrow that connects the tail of the first vector with the head of the last vector.

2. No, the order of adding vectors has no effect on the resultant.

3. Scale: Let 1 cm = 10 N



4. Textbook question 1.b:



Scale : 1 cm = 2 km

(5.5 cm = 11 km)

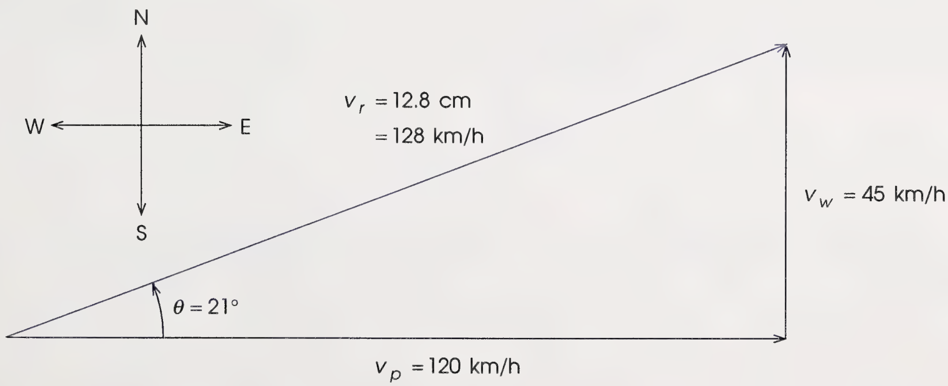
The resultant displacement is 15.6 km, E 45° N.

Notes:

- The solution begins with a statement about directions and a scale.
- The direction can be expressed as either 45° north of east or E 45° N, which means "from east go 45° north".

Textbook question 5:

Scale: 1 cm = 10 km/h



The resultant velocity is 128 km/h, E 21° N.

5.

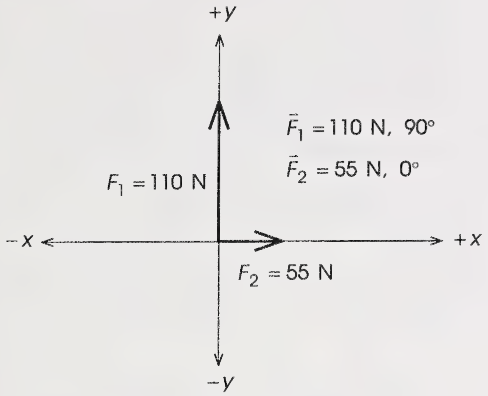
Proper Use of Vector Notation in Physics 30	
Operation	**Is the vector notation consistent with what is expected in Physics 30?**
$3\vec{A}$	Yes, this notation is consistent. It is acceptable to show a constant multiplied by a vector.
$\vec{A}+\vec{B}$	Yes, this notation is consistent. It is acceptable to show the addition of two vectors.
$\vec{A}\times\vec{C}$	Yes, this notation is consistent. Although vectors are normally not multiplied by other vectors, the vectors shown here are colinear, so it's like multiplying a negative scalar by a positive one.
$-359\vec{D}$	Yes, this notation is consistent. It is acceptable to show a constant multiplied by a vector.
$\vec{C}-\vec{D}$	Yes, this notation is consistent. It is acceptable to show the subtraction of two vectors.
$\dfrac{\vec{B}}{\vec{C}}$	No, this notation is inconsistent. It is beyond the scope of Physics 30 to show a division between vectors such as $\vec{B}$ and $\vec{C}$.
$m\vec{B}+n\vec{C}$	Yes, this notation is consistent. It is acceptable to show the addition of two vectors which are each multiplied by a scalar.

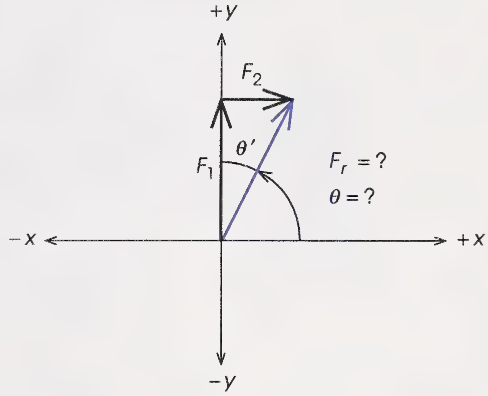
6. The proper way to show the calculation of the tangent of θ is without vector notation since the operation involves the division of quantities. The following methods would be more correct:

$$\tan\,\theta=\frac{\left|\vec{A}\right|}{\left|\vec{B}\right|} \quad \text{or} \quad \tan\,\theta=\frac{A}{B}$$

The magnitude of the vector can be expressed with absolute value signs or by writing the quantity without the vector sign.

7. Textbook question 7:





- Find the magnitude of the resultant.

$$C^2 = A^2 + B^2$$
$$(F_r)^2 = (F_1)^2 + (F_2)^2$$
$$= (110\ \text{N})^2 + (55\ \text{N})^2$$
$$= 15\,125\ \text{N}^2$$
$$F_r = 123\ \text{N}$$
$$= 1.2 \times 10^2\ \text{N}$$

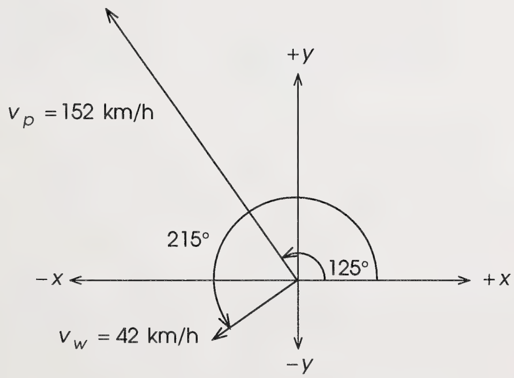
- Find the direction of the resultant.

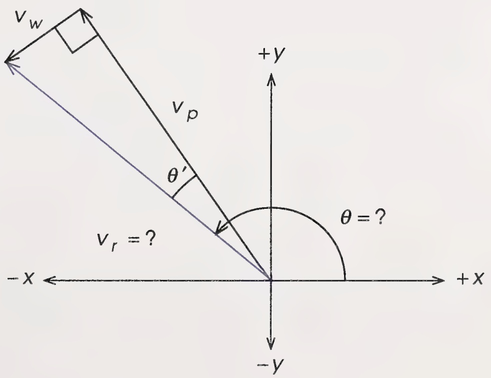
$$\tan\theta' = \frac{F_2}{F_1}$$
$$= \frac{55\ \text{N}}{110\ \text{N}}$$
$$= 0.50$$
$$\theta' = 26.57°$$

$$\theta = 90° - \theta'$$
$$= 90° - (26.57°)$$
$$= 63.4°$$
$$= 63°$$

$$\vec{F}_r = 1.2 \times 10^2\ \text{N, } 63°$$

Textbook question 10:





- Find the magnitude of the resultant.

$$C^2 = A^2 + B^2$$
$$(v_r)^2 = (v_p)^2 + (v_w)^2$$
$$= (152\ \text{km/h})^2 + (42\ \text{km/h})^2$$
$$= 24\,868\ \text{km}^2/\text{h}^2$$
$$v_r = 157.7\ \text{km/h}$$
$$= 1.6 \times 10^2\ \text{km/h}$$

- Find the direction of the resultant.

$$\tan\theta' = \frac{v_w}{v_p}$$
$$= \frac{42\ \text{km/h}}{152\ \text{km/h}}$$
$$= 0.2763$$
$$\theta' = 15.4°$$

$$\theta = \theta' + (\text{angle of } \vec{v}_p)$$
$$= 15.4° + 125°$$
$$= 140.4°$$
$$= 1.4 \times 10^2{}°$$

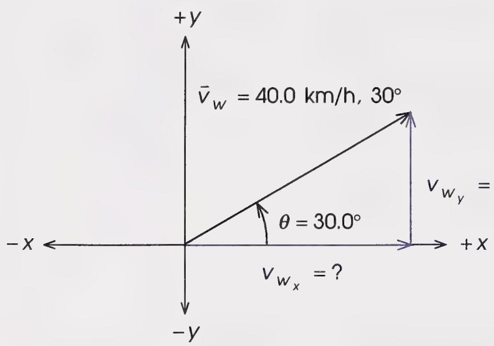
$$\vec{v}_r = 1.6 \times 10^2\ \text{km/h},\ 1.4 \times 10^2{}°$$

8. $\sin\theta = \dfrac{F_v}{F}$ $\cos\theta = \dfrac{F_h}{F}$

$F_v = F(\sin\theta)$ $F_h = F(\cos\theta)$

Since trigonometry requires quantities to be multiplied and divided, the vector notation should be dropped.

9.



- Find the x-component.

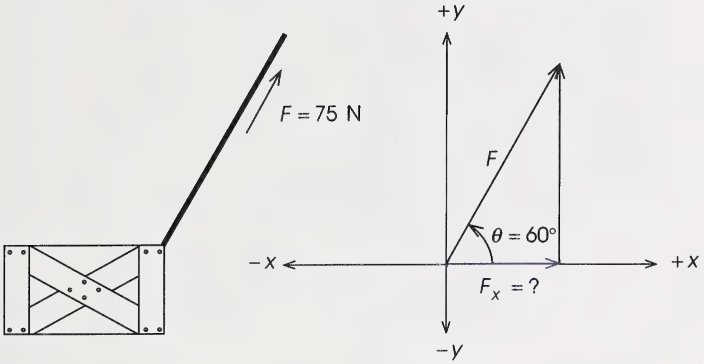
$$v_{w_x} = v_w(\cos\theta)$$
$$= (40.0\ \text{km/h})(\cos 30°)$$
$$= 34.6\ \text{km/h}$$

- Find the y-component.

$$v_{w_y} = v_w(\sin\theta)$$
$$= (40.0\ \text{km/h})(\sin 30°)$$
$$= 20.0\ \text{km/h}$$

The components can be treated as scalars since the direction is already implied by the x-y axis.

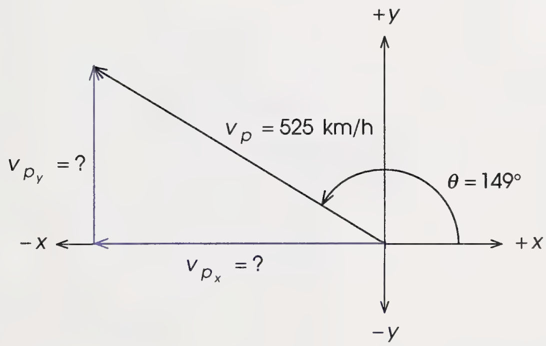
10. Textbook question 11:



$$
\begin{aligned}
F_x &= F(\cos\theta) \\
&= (75\ \text{N})(\cos 60^\circ) \\
&= 37.5\ \text{N} \\
&= 38\ \text{N}
\end{aligned}
$$

The problem becomes much easier to solve if the standard *x*-*y* axis is used.

Textbook question 12:



Textbook question 12.a.:

Find the *y*-component.

$$
\begin{aligned}
v_{p_y} &= v_p(\sin\theta) \\
&= (525\ \text{km/h})(\sin 149^\circ) \\
&= 270\ \text{km/h}
\end{aligned}
$$

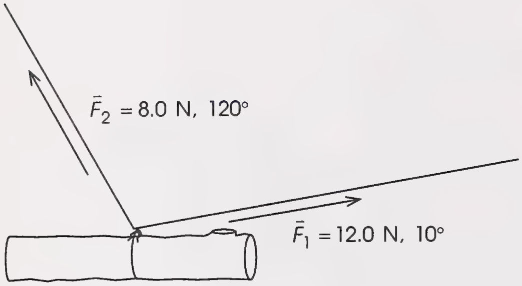
Textbook question 12.b.:

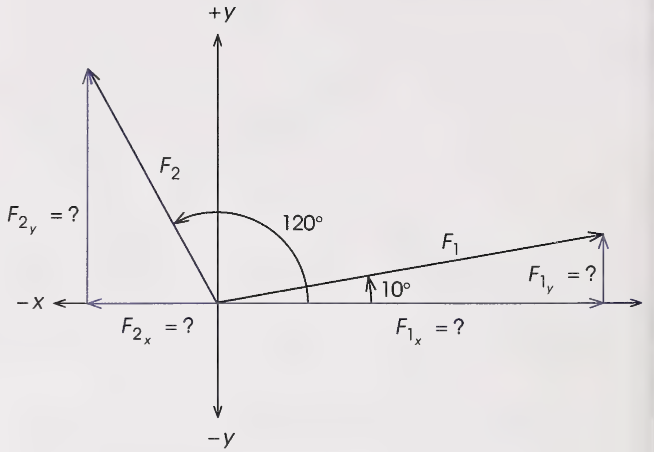
Find the *x*-component.

$$
\begin{aligned}
v_{p_x} &= v_p(\cos\theta) \\
&= (525\ \text{km/h})(\cos 149^\circ) \\
&= -450\ \text{km/h}
\end{aligned}
$$

Using the *x*-*y* axis removes any ambiguity about the meaning of negatives and positives. When the angles are measured counterclockwise from the *x*-axis, sine and cosine functions will automatically look after positive and negative values.

11.





Step 1: Resolve F_1.

$$\begin{aligned} F_{1_x} &= F_1(\cos\theta) \\ &= (12.0\ \text{N})(\cos 10°) \\ &= 11.8\ \text{N} \end{aligned}$$

$$\begin{aligned} F_{1_y} &= F_1(\sin\theta) \\ &= (12.0\ \text{N})(\sin 10°) \\ &= 2.08\ \text{N} \end{aligned}$$

Step 2: Resolve F_2.

$$\begin{aligned} F_{2_x} &= F_2(\cos\theta) \\ &= (8.0\ \text{N})(\cos 120°) \\ &= -4.0\ \text{N} \end{aligned}$$

$$\begin{aligned} F_{2_y} &= F_2(\sin\theta) \\ &= (8.0\ \text{N})(\sin 120°) \\ &= 6.93\ \text{N} \end{aligned}$$

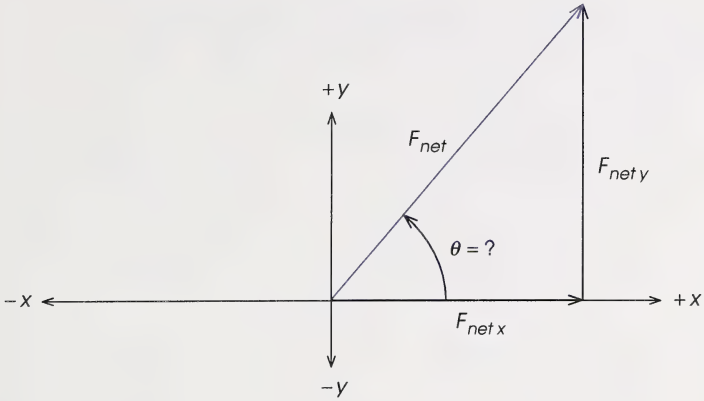
Step 3: Find the magnitude of the resultant.

$$\begin{aligned} F_{net\,x} &= (11.8\ \text{N}) + (-4.0\ \text{N}) \\ &= 7.8\ \text{N} \end{aligned}$$

$$\begin{aligned} F_{net\,y} &= (2.08\ \text{N}) + (6.93\ \text{N}) \\ &= 9.01\ \text{N} \end{aligned}$$

$$\begin{aligned} C^2 &= A^2 + B^2 \\ F_{net}{}^2 &= (F_{net\,x})^2 + (F_{net\,y})^2 \\ &= (7.8\ \text{N})^2 + (9.01\ \text{N})^2 \\ &= 142.0\ \text{N}^2 \\ F_{net} &= 11.9\ \text{N} \\ &= 12\ \text{N} \end{aligned}$$

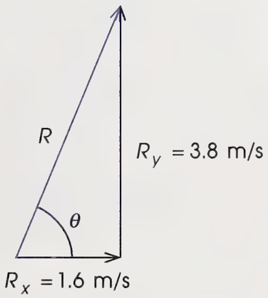
Step 4: Find the direction of the resultant.



$$\begin{aligned}\tan\theta &= \frac{F_{net\,y}}{F_{net\,x}}\\ &= \frac{9.01\text{ N}}{7.8\text{ N}}\\ &= 1.155\\ \theta &= 49.1^\circ\\ &= 49^\circ\end{aligned}$$

The net force on the log is 12 N, 49°.

12. Textbook question 17. a.:



$$\begin{aligned}R &= \sqrt{R_y{}^2 + R_x{}^2}\\ &= \sqrt{(3.8\text{ m/s})^2 + (1.6\text{ m/s})^2}\\ &= 4.1\text{ m/s}\end{aligned}$$

$$\begin{aligned}\tan\theta &= \frac{3.8\text{ m/s}}{1.6\text{ m/s}}\\ &= 2.375\\ \theta &= 67^\circ\text{ downstream}\end{aligned}$$

Kym's boat will be travelling 4.1 m/s, 67° downstream.

Textbook question 17. b.:

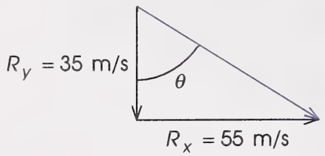
$$\begin{aligned}t &= \frac{d}{v}\\ &= \frac{240\text{ m}}{3.8\text{ m/s}}\\ &= 63\text{ s}\end{aligned}$$

Remember that you need to use the velocity and displacement which are in the same direction across the river.

Textbook question 17. c.:

$$\begin{aligned} d &= vt \\ &= (1.6\ \text{m/s})(63\ \text{s}) \\ &= 1.0 \times 10^{2}\ \text{m} \end{aligned}$$

Textbook question 19. a.:

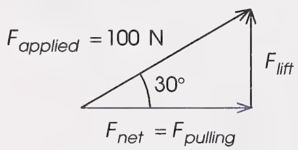


$$\begin{aligned} R &= \sqrt{R_y{}^2 + R_x{}^2} \\ &= \sqrt{(35\ \text{m/s})^2 + (55\ \text{m/s})^2} \\ &= 65\ \text{m/s} \end{aligned}$$

Textbook question 19. b.:

$$\begin{aligned} \tan\ \theta &= \frac{55\ \text{m/s}}{35\ \text{m/s}} \\ &= 1.5714 \\ \theta &= 58^\circ\ \text{from the vertical} \end{aligned}$$

Textbook question 23. a.:



$$\begin{aligned} F_{net} &= (100\ \text{N})(\cos\ 30^\circ) \\ &= 86.6\ \text{N} \end{aligned}$$

The acceleration of the crate is produced by the pulling force, not the lifting force. The pulling force is the net force.

$$\begin{aligned} F_{net} &= ma \\ a &= \frac{F_{net}}{m} \\ &= \frac{86.6\ \text{N}}{40\ \text{kg}} \\ &= 2.2\ \text{m/s}^2 \end{aligned}$$

Textbook question 23. b.:

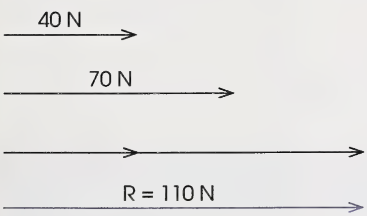
Since the crate is vertically stable, the upward forces must equal the downward forces. The only downward force is gravity. The two upward forces are the lift force and the force exerted by the ice, the normal force.

$$\begin{aligned} F_g &= mg \\ &= (40\ \text{kg})(9.81\ \text{N/kg}) \\ &= 3.92 \times 10^2\ \text{N} \\ &= 3.9 \times 10^2\ \text{N} \end{aligned}$$

$$\begin{aligned} F_{lift} &= F_A(\sin\theta) \\ &= (100\ \text{N})(\sin 30°) \\ &= 50\ \text{N} \end{aligned}$$

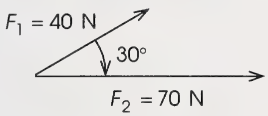
$$\begin{aligned} \vec{F}_{net} &= \vec{F}_{lift} + \vec{F}_g + \vec{F}_{normal} \\ F_{net} &= 0 \\ 0 &= +|F_{lift}| - |F_g| + |F_{normal}| \\ |F_{normal}| &= |F_g| - |F_{lift}| \\ &= (392.4\ \text{N}) - (50\ \text{N}) \\ &= 342.4\ \text{N} \\ &= 3.4 \times 10^2\ \text{N} \end{aligned}$$

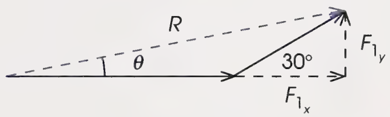
Textbook question 26. a.:



$$\begin{aligned} R &= (40\ \text{N}) + (70\ \text{N}) \\ &= 110\ \text{N} \end{aligned}$$

Textbook question 26. b.:





Calculations for the 40-N force:

$$\begin{aligned} F_{1_x} &= (40\ \text{N})(\cos 30°) \\ &= 34.6\ \text{N} \end{aligned}$$

$$\begin{aligned} F_{1_y} &= (40\ \text{N})(\sin 30°) \\ &= 20.0\ \text{N} \end{aligned}$$

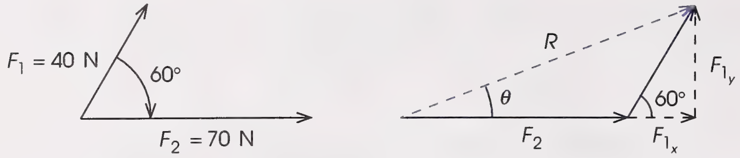
Add the components to get the resultant x- and y- components.

$$\begin{aligned} R_x &= (70\ \text{N}) + (34.6\ \text{N}) \\ &= 104.6\ \text{N} \end{aligned}$$

$$\begin{aligned} R_y &= (0\ \text{N}) + (20\ \text{N}) \\ &= 20\ \text{N} \end{aligned}$$

$$\begin{aligned} R &= \sqrt{R_x^{\ 2} + R_y^{\ 2}} \\ &= \sqrt{(104.6\ \text{N})^2 + (20.0\ \text{N})^2} \\ &= 1.1 \times 10^2\ \text{N} \end{aligned}$$

Textbook question 26. c.:



Calculations for the 40-N force:

$$F_{1_x} = (40\ \text{N})(\cos 60°)$$
$$= 20.0\ \text{N}$$

$$F_{1_y} = (40\ \text{N})(\sin 60°)$$
$$= 34.6\ \text{N}$$

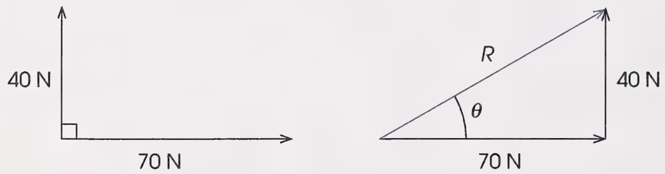
Add the components to get the resultant x- and y-components.

$$R_x = (70\ \text{N}) + (20\ \text{N})$$
$$= 90\ \text{N}$$

$$R_y = (0\ \text{N}) + (34.6\ \text{N})$$
$$= 34.6\ \text{N}$$

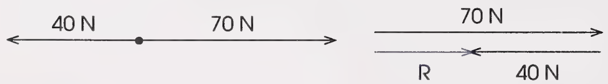
$$R = \sqrt{R_x{}^2 + R_y{}^2}$$
$$= \sqrt{(90.0\ \text{N})^2 + (34.6\ \text{N})^2}$$
$$= 96\ \text{N}$$

Textbook question 26. d.:



$$R = \sqrt{R_x{}^2 + R_y{}^2}$$
$$= \sqrt{(40\ \text{N})^2 + (70\ \text{N})^2}$$
$$= 81\ \text{N}$$

Textbook question 26. e.:

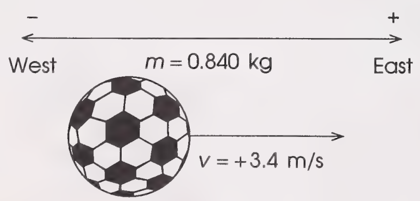


$$R = (70\ \text{N}) - (40\ \text{N})$$
$$= 30\ \text{N}$$

Section 1: Activity 2

1. The impulse given to an object is equal to the change in its momentum. This is demonstrated on pages 176 and 177 of the textbook through the use of Newton's second law.

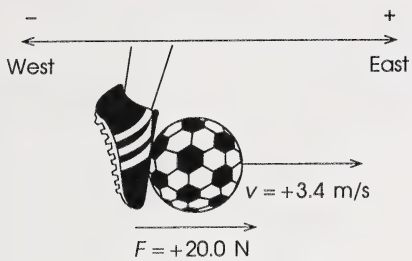
2. a.



$$\vec{p} = m\vec{v}$$
$$= (0.840\ \text{kg})(+3.4\ \text{m/s})$$
$$= 2.9\ \text{kg} \bullet \text{m/s}$$

The positive value for the momentum indicates that it is going east.

b.



$$\vec{F}\Delta t = (+20.0\ \text{N})(0.220\ \text{s})$$
$$= 4.40\ \text{N} \bullet \text{s}$$

The positive value for the impulse indicates that it is going east.

c. $$\vec{F}\Delta t = m\Delta\vec{v}$$
$$\vec{F}\Delta t = m\left(\vec{v}_f - \vec{v}_i\right)$$
$$\vec{v}_f = \frac{\vec{F}\Delta t}{m} + \vec{v}_i$$
$$= \left(\frac{+4.40\ \text{N} \bullet \text{s}}{0.840\ \text{kg}}\right) + (+3.4\ \text{m/s})$$
$$= 8.6\ \text{m/s}$$

The positive value for the final velocity indicates that it is going east.

3.

Newton's Law	Main Idea	How It Relates to Momentum
First	An object with no net force acting on it will move with a constant velocity.	Since mass is usually constant, when no net force acts on an object, it will maintain a constant momentum.
Second	The acceleration of a body is directly proportional to the net force acting on it and inversely proportional to the mass of the body.	When a net force acts and the body changes velocity, its momentum must also change.

4. One advantage to using $\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$ would be in the case of a body changing its mass while in motion. In such a case $\vec{F} = m\vec{a}$ would not provide a simple and clear indication of the resulting motion since a constant force would not provide a constant acceleration. Such a case might be found in the resulting motion of a rocket which loses mass as it burns its fuel.

5. a.

$$\vec{F} = m\vec{a}$$

$$\vec{F} = m\left(\frac{\Delta\vec{v}}{\Delta t}\right)$$

$$\vec{F} = \frac{m\Delta\vec{v}}{\Delta t}$$

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$$

$$\vec{F}\Delta t = \Delta\vec{p}$$

b. An alternative solution would be to determine the average acceleration of the baseball first. Then, using Newton's second law, the force could be determined.

$$\begin{aligned}\vec{a} &= \frac{\vec{v}_f - \vec{v}_i}{\Delta t}\\ &= \frac{(-38\ \text{m/s} - 38\ \text{m/s})}{8.0\times10^{-4}\ \text{s}}\\ &= -9.5\times10^{4}\ \text{m/s}^2\end{aligned}$$

$$\begin{aligned}\vec{F} &= m\vec{a}\\ &= (0.144\ \text{kg})\left(-9.5\times10^{4}\ \text{m/s}^2\right)\\ &= -1.4\times10^{4}\ \text{N}\end{aligned}$$

Neither method is better. Both provide the same answer with a clear indication of direction. Note that one of the calculations in the text has an error in the exponent.

6. Textbook question 2:

The solution can be found on page 669 of the textbook.

Textbook question 3:

The solution can be found on page 669 of the textbook.

7. Textbook question 4:

The solution can be found on page 669 of the textbook.

8. a. Due to Newton's third law of motion, the graphs would be identical. Since the two graphs are identical, the magnitude of the impulse given to the ball must be the same as the magnitude of the impulse given to the box. However, because the two forces are in opposite directions, the impulse could be said to be equal in magnitude but opposite in direction.

b. The change in momentum of the ball must be equal in magnitude but opposite in direction to the momentum change of the box.

9. Since Nicole's speed on impact is the same in each case, and the final speed for each jump is zero, the change in momentum in both scenarios is equal. Therefore, since $\Delta\vec{p} = \vec{F}\Delta t$, the impulse given to Nicole in each case is the same.

10. Jumping into hay:

$$\vec{F}\Delta t = m\Delta\vec{v}$$

$$\begin{aligned}\vec{F} &= \frac{m\Delta\vec{v}}{\Delta t}\\ &= \frac{(50.0\ \text{kg})(0\ \text{m/s} - 3.00\ \text{m/s})}{1.130\ \text{s}}\\ &= -133\ \text{N}\end{aligned}$$

Jumping into rocks:

$$\begin{aligned}\vec{F} &= \frac{m\Delta\vec{v}}{\Delta t}\\ &= \frac{(50.0\ \text{kg})(0\ \text{m/s} - 3.00\ \text{m/s})}{0.245\ \text{s}}\\ &= -612\ \text{N}\end{aligned}$$

These calculations show that Nicole exerts a greater force on the rocks. The reason that the rocks are more painful has to do with Newton's third law. If Nicole exerts a downward force of 612 N on the rocks, the rocks exert an upward force of 612 N on her. It's the force of the rocks on Nicole that causes the injuries.

11. A bungee cord is basically designed to reduce the downward velocity and momentum of your body to zero without breaking bones or damaging muscles. The impulse provided by the cord must equal the change in momentum of your body $(F\Delta t = m\Delta v)$. The cord must gradually apply the force over a relatively long time interval. The cord does this by stretching – the greater the stretch, the greater the force.

The mass of the jumper will influence the magnitude of the change in momentum and the impulse that must be provided by the cord. This is why most bungee jump operators require a weigh-in before jumping. This allows them to accurately match the right cord (impulse) to each individual jumper (change in momentum).

At the lowest point of the descent, the original gravitational potential energy of the jumper has been partly stored in the stretching of the cord and has partly dissipated as thermal energy. Although the jumper will tend to bob up and down, the cord is designed to eventually dissipate all the energy so the jumper can be untied from below.

12. Textbook question 2:

You should move your hands in the same direction as the ball is travelling to increase the time of the collision and reduce the force.

13. Textbook question 1.1:

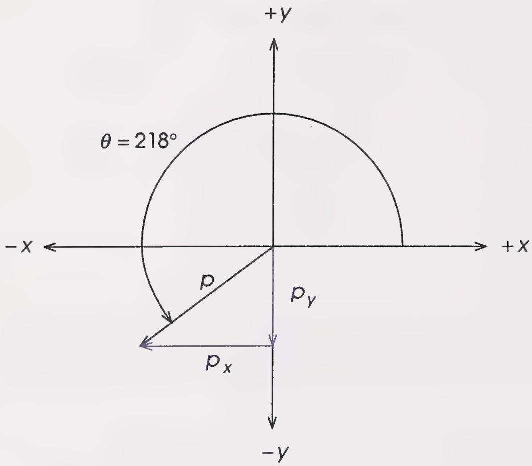
The momentum is different only because the cars are travelling in different directions.

14. a.

$$\begin{aligned} m &= 434\ \text{g} = 0.434\ \text{kg}\\ \vec{v} &= 5.8\ \text{m/s},\ 218^\circ\\ v &= 5.8\ \text{m/s}\\ p &= ?\end{aligned}$$

$$\begin{aligned}\vec{p} &= m\vec{v}\\ p &= mv\\ &= (0.434\ \text{kg})(5.8\ \text{m/s})\\ &= 2.5\ \text{kg}\bullet\text{m/s}\end{aligned}$$

b.



c. This question is answered on the previous diagram.

d. $\theta = 218°$

$p = 2.5\ \text{kg} \bullet \text{m/s}$

$p_x = ?$

$p_y = ?$

$$\begin{aligned} p_x &= p(\cos\theta) \\ &= (2.5\ \text{kg} \bullet \text{m/s})(\cos 218°) \\ &= -1.97\ \text{kg} \bullet \text{m/s} \\ &= -2.0\ \text{kg} \bullet \text{m/s} \end{aligned}$$

$$\begin{aligned} p_y &= p(\sin\theta) \\ &= (2.5\ \text{kg} \bullet \text{m/s})(\sin 218°) \\ &= -1.54\ \text{kg} \bullet \text{m/s} \\ &= -1.5\ \text{kg} \bullet \text{m/s} \end{aligned}$$

Notes:

- Solving for *x*- and *y*-components does not involve vector notation since the trigonometry ratios are based on magnitudes.
- Since the momentum vector is in the third quadrant, both the *x*- and *y*-components are negative.

Section 1: Follow-up Activities

Extra Help

1. The vector quantities are momentum, force, acceleration, displacement, impulse, and velocity.

   The scalar quantities are temperature, speed, age, mass, distance, and time.

2. The two equations needed are $p_x = p(\cos\theta)$ and $p_y = p(\sin\theta)$, assuming the magnitude of the momentum is p and the direction is θ (angles measured from 0° at the positive *x*-axis).

3. Textbook question 16:

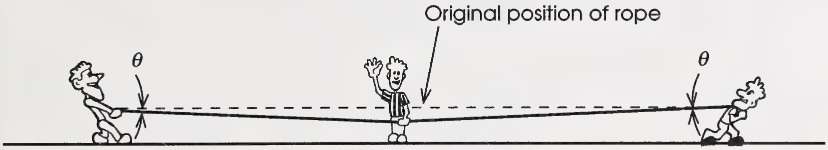
   The solution can be found on page 665 of your textbook. Note the changes from the recommended notation.

4.

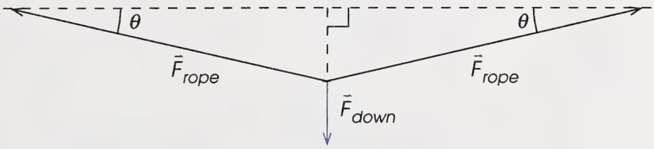
Quantity	Symbol	Equation	Units
Force	$\vec{F}$	$\vec{F} = m\vec{a}$	$N = kg \bullet m/s^2$
Impulse	$\vec{F}\Delta t$	$\vec{F}\Delta t = \Delta\vec{p}$	$N \bullet s = kg \bullet m/s$
Momentum	$\vec{p}$	$\vec{p} = m\vec{v}$	$kg \bullet m/s$

Enrichment

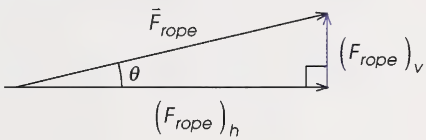
1. Diagram of rope:



Vector diagram:



- The force exerted on each side by the rope can be broken down into horizontal and vertical components.



- As shown in the force diagram, the vertical component of the force supplied by the rope is very small when compared to full force of the rope. It is this vertical component that helps to balance the downward force applied by the person in the middle. This is why it takes huge forces exerted by the people at each end to counteract a relatively small force applied to the middle. In fact, when the angle (θ) is very small, it is impossible for the nearly horizontal rope forces to overcome even the smallest downward forces.

2. Check the answers given at the back of the booklet *Supplemental Lessons*.

3. First, having a midsole that cushions some of the force works by increasing the time interval for the change of momentum. By increasing Δt, there must be a corresponding decrease in the force (F) so that $F\Delta t$ remains constant. Secondly, having two sets of air cylinders will provide shock absorption where individual runners need it most, since each runner's foot is different.

Section 2: Activity 1

1. a. The friction on the air table is greatly reduced, so there is virtually no net horizontal force. As well, the vertical forces of gravity and air on the puck are balanced. Therefore, the system is very nearly isolated. Since no significant amount of matter enters or leaves, it can also be considered to be closed.

   b. Textbook question 3:

   No net force implies no net impulse $(F\Delta t)$, and thus no net change in momentum for the system. However, individual parts of the system may experience changes in their individual momentum values, but the total momentum should stay constant.

2. Without friction, the system, including the ball, bat, and batter, becomes more isolated. Thus the bat and batter would end up moving in the original direction of the ball after the ball is hit.

3. In a collision between two objects (A and B), Newton's third law says that $\vec{F}_{AB} = -\vec{F}_{BA}$. Since the time of impact is the same for both objects,

   $\vec{F}_{AB}\Delta t = -\vec{F}_{BA}\Delta t$ and thus

   $\Delta\vec{p}_B = -\Delta\vec{p}_A$

   $\vec{p}_B{}' - \vec{p}_B = -\left(\vec{p}_A{}' - \vec{p}_A\right)$ which leads directly to

   $\vec{p}_A + \vec{p}_B = \vec{p}_A{}' + \vec{p}_B{}'$

   This is the algebraic expression for the law of conservation of momentum.

4. a. $\Delta\vec{p}_A + \Delta\vec{p}_B = 0$ b. $\vec{p}_A + \vec{p}_B = \vec{p}_A{}' + \vec{p}_B{}'$ or $\sum\vec{p}_{before} = \sum\vec{p}_{after}$

Section 2: Activity 2

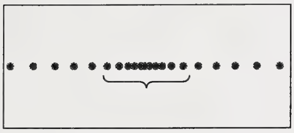
1. The answers to this question will vary. The following sample represents one approach.

   Materials: air table with air pucks, puck launcher (if available), piece of newsprint, velcro strips or putty to wrap around the pucks

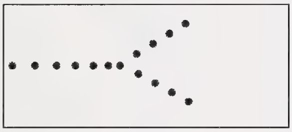
   Procedure:

   - Read each step carefully.
   - Review all the recommended safety procedures with the lab instructor.
   - Wrap the velcro strips or putty around each puck. Make sure that they do not extend below the bottom of the puck.
   - Measure the mass of each puck before or after performing the experiment.

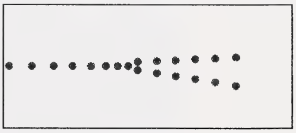
- Make sure the table is level before proceeding by placing a puck in the centre and turning on the air. When the table is level, the puck should be relatively motionless.
- Place one of the pucks in the centre of the sheet of newsprint on the air table.
- Turn on the air and spark timer, which should be set at 20-ms intervals (or any other appropriate setting if 20 ms is not available).
- Launch or push the second puck directly at the stationary puck.
- A few seconds after the collision, stop the timer.
- At this point, you should turn the power off and analyse the record of dots on the sheet of newsprint. It is all right if the pucks rotated slightly because the dots are clearly separated. However, if too much rotation occurs, it is not truly a linear collision.



Perfectly linear, but dots overlap, making it difficult to analyse



Too much rotation and not a linear collision



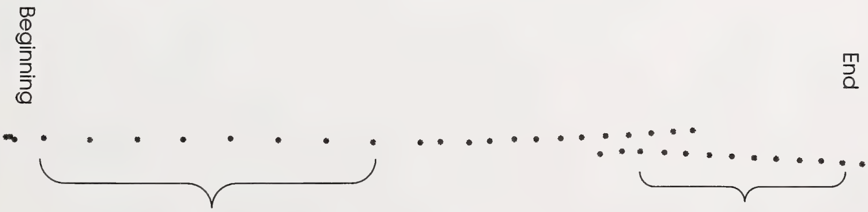
Only slightly off-line, but easiest to analyse and get good results from

- It is extremely important that you recognize the portion of the dots that represent uniform motion. If the spacing looks irregular, the collision has likely occurred.

2. Answers will vary. See question 4 and its solution for a sample answer.

3. Answers will vary. See question 5 and its solution for a sample answer.

4. Answers to this question will vary depending on the operation of the air table that you used. The following data was taken from the tear-out sheet provided at the end of the Appendix. Note that this diagram is not to scale.

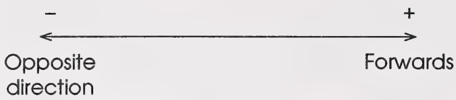


$d = 9.1$ cm
$t = 7 \times 20$ ms
$= 140$ ms

$d = 5.6$ cm
$t = 9 \times 20$ ms
$= 180$ ms

You may have chosen different intervals and arrived at different measurements.

Since you are dealing with vectors, a sign convention is necessary. You would likely choose positive as being the forward direction, since there is no motion in the opposite direction.



The data was then interpreted as shown in the following chart. You may have taken different readings.

	Before Collision Puck A	After Collision Pucks A and B
Displacement (cm)	+ 9.1	+ 5.6
Number of Intervals	7	9
Time (s)	0.140	0.180
Velocity (cm/s)	+ 65	+ 31
Mass (kg)	0.550	1.100
Momentum (kg • cm/s)	36	34

Sample equations and calculations follow.

$$\begin{aligned}\vec{v} &= \frac{\vec{d}}{t}\\ &= \frac{+9.1\ \text{cm}}{0.140\ \text{s}}\\ &= +65\ \text{cm/s}\end{aligned}$$

$$\begin{aligned}\vec{v}\,' &= \frac{\vec{d}}{t}\\ &= \frac{+5.6\ \text{cm}}{0.180\ \text{s}}\\ &= +31\ \text{cm/s}\end{aligned}$$

$$\begin{aligned}\vec{p} &= m\vec{v}\\ &= (0.550\ \text{kg})(+65\ \text{cm/s})\\ &= +36\ \text{kg} \bullet \text{cm/s}\end{aligned}$$

$$\begin{aligned}\vec{p}\,' &= m\vec{v}\,'\\ &= (1.100\ \text{kg})(+31\ \text{cm/s})\\ &= +34\ \text{kg} \bullet \text{cm/s}\end{aligned}$$

5. $\text{actual} = 35.75\ \text{kg} \bullet \text{cm/s}$
$\text{experimental} = 34.22\ \text{kg} \bullet \text{cm/s}$

$$\begin{aligned}\text{percent error} &= \frac{\text{actual} - \text{experimental}}{\text{actual}} \times 100\\ &= \frac{(35.75\ \text{kg} \bullet \text{cm/s}) - (34.22\ \text{kg} \bullet \text{cm/s})}{35.75\ \text{kg} \bullet \text{cm/s}} \times 100\%\\ &= 4.0\%\end{aligned}$$

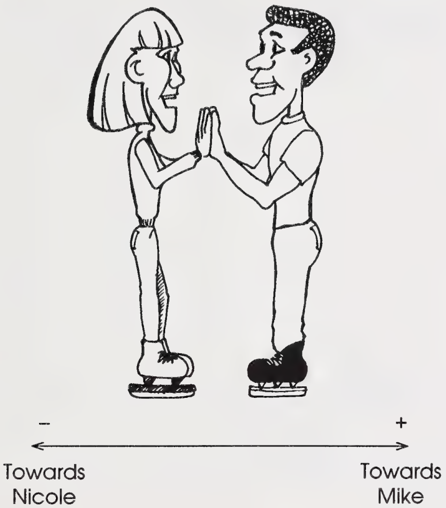
The initial and final magnitudes of the momenta in this experiment were extremely close, even though the velocities were not. This allows you to make a confident conclusion that momentum is conserved.

Section 2: Activity 3

1. a. Since neither of them are moving, their initial momentum (total) is zero.

   b. Yes, momentum should be conserved because, with a small force of friction, they could be considered an isolated system. After they push off they should still have a total momentum of zero.

   c. Nicole will have a greater speed because of her smaller mass. The proof follows.



$\bar{p}_N = \bar{p}_M = 0$ before pushing

$\bar{p}_N{}'$ $\quad\bar{p}_M{}'$ after pushing

$\longleftarrow \quad \longrightarrow$

$$\sum \bar{p}_{before} = \sum \bar{p}_{after}$$
$$\bar{p}_N + \bar{p}_M = \bar{p}_N{}' + \bar{p}_M{}'$$
$$0 = \bar{p}_N{}' + \bar{p}_M{}'$$
$$\bar{p}_M{}' = -\bar{p}_N{}'$$

From this statement, it is obvious that the magnitudes are equal, but the directions are not.

$$\left|\bar{p}_M{}'\right| = \left|\bar{p}_N{}'\right|$$
$$m_M v_M{}' = m_N v_N{}'$$
$$\frac{m_M}{m_N} = \frac{v_N{}'}{v_M{}'}$$

Since $\frac{m_M}{m_N} > 1$, $\frac{v_N{}'}{v_M{}'} > 1$, and so $v_N{}' > v_M{}'$.

2.

Type of Collision	Distinguishing Feature	Equation
Hit-and-stick	After collision, the two (or more) bodies are joined.	$m_A\vec{v}_A + m_B\vec{v}_B = (m_A + m_B)\vec{v}'$
Explosion	One large object breaks into two or more objects. Often the initial momentum is zero and the magnitude of the momenta of each final object is equal.	$0 = m_A\vec{v}_A' + m_B\vec{v}_B'$ or $(m_A + m_B)\vec{v} = m_A\vec{v}_A' + m_B\vec{v}_B'$

3. Textbook question 5:

The solution is found on page 669 of the textbook.

Textbook question 6:

The solution is found on page 670 of the textbook.

Textbook question 7:

The solution is found on page 670 of the textbook.

4. Textbook question 8:

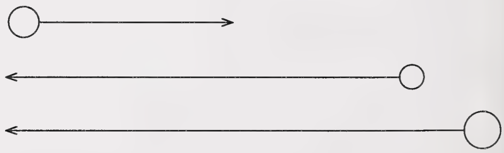
– $\longleftrightarrow$ +

Opposite direction — Forwards

$m_A = 0.50$ kg — $m_B = 1.00$ kg

$\vec{v}_A = +6.0$ m/s — $\vec{v}_B = -12.0$ m/s

$\vec{v}_A' = -14$ m/s — $\vec{v}_B' = ?$



$$\sum \vec{p}_{before} = \sum \vec{p}_{after}$$

$$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}_A' + m_B\vec{v}_B'$$

$$\vec{v}_B' = \frac{m_A\vec{v}_A + m_B\vec{v}_B - m_A\vec{v}_A'}{m_B}$$

$$= \frac{(0.50\ \text{kg})(+6.0\ \text{m/s}) + (1.00\ \text{kg})(-12.0\ \text{m/s}) - (0.50\ \text{kg})(-14\ \text{m/s})}{1.00\ \text{kg}}$$

$$= -2.0\ \text{m/s}$$

The second ball continues in the negative direction at 2.0 m/s.

5. Textbook questions 9:

The solution is found on page 670 in the textbook.

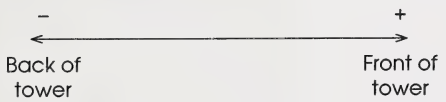
Textbook questions 10:

The solution is found on page 670 in the textbook.

Textbook questions 11:

The solution is found on page 670 in the textbook.

6. Textbook question 12. a.:



$$m_{cannon} = 225 \text{ kg}$$
$$m_{ball} = 4.5 \text{ kg}$$
$$\vec{d}_{ball}{}' = +215 \text{ m}$$
$$\vec{d}_{cannon}{}' = ?$$

$$\sum \vec{p}_{before} = \sum \vec{p}_{after}$$
$$m_c\vec{v}_c + m_b\vec{v}_b = m_c\vec{v}_c{}' + m_b\vec{v}_b{}'$$

But $\vec{v}_c = \vec{v}_b = 0$, so $0 = m_c\vec{v}_c{}' + m_b\vec{v}_b{}'$, and the horizontal velocities are related to the horizontal displacements by $\vec{v}_c{}' = \frac{\vec{d}_c{}'}{t_c}$ and $\vec{v}_b{}' = \frac{\vec{d}_b{}'}{t_b}$, thus $0 = m_c\left(\frac{\vec{d}_c{}'}{t_c}\right) + m_b\left(\frac{\vec{d}_b{}'}{t_b}\right)$.

Since $t_c = t_b$, you can call it t.

$$0 = \frac{m_c\vec{d}_c{}'}{t} + \frac{m_b\vec{d}_b{}'}{t}.$$

When rearranged, the equation can be used as shown.

$$\begin{aligned}
\vec{d}_c{}' &= \frac{-m_b\vec{d}_b{}'}{m_c} \\
&= \frac{-(4.5 \text{ kg})(+215 \text{ m})}{225 \text{ kg}} \\
&= -4.3 \text{ m}
\end{aligned}$$

The cannon will land 4.3 m from the back of the tower.

Textbook question 12. b.:

The cannon wheels are frictionless, so the cannon does not lose speed as it crosses the tower. Its horizontal speed will not decrease.

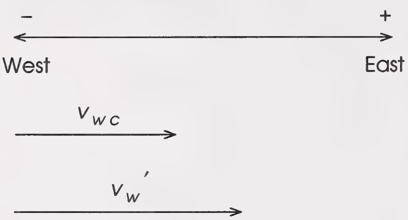
7. Textbook question 7:

   Momentum is conserved. The change in momentum experienced by the gases must be balanced by an equal change in momentum by the spacecraft.

   Textbook question 10:

   The brakes stop the wheels from moving. The external force is the force of friction exerted by the road. In icy conditions this force is reduced, so the car will not stop as easily.

8. Textbook question 22:



intial $\vec{v}$ of woman and cart $(\vec{v}_{wc}) = +5.0\ \text{m/s}$

final $\vec{v}$ of woman $(\vec{v}_w') = +7.0\ \text{m/s}$

mass of woman $(m_w) = 50\ \text{kg}$

mass of cart $(m_c) = 10\ \text{kg}$

final $\vec{v}$ of cart $(\vec{v}_c') = ?$

$$\sum \vec{p}_{before} = \sum \vec{p}_{after}$$

$$(m_w + m_c)\vec{v}_{wc} = m_w\vec{v}_w' + m_c\vec{v}_c'$$

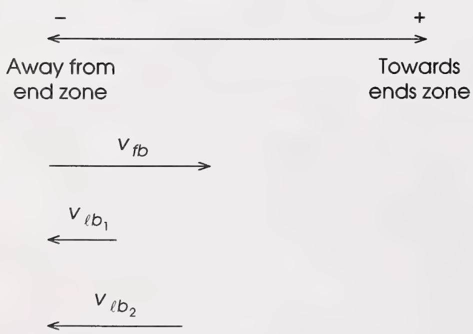
$$\vec{v}_c' = \frac{(m_w + m_c)\vec{v}_{wc} - m_w\vec{v}_w'}{m_c}$$

$$= \frac{(50\ \text{kg} + 10\ \text{kg})(+5.0\ \text{m/s}) - (50\ \text{kg})(+7.0\ \text{m/s})}{10\ \text{kg}}$$

$$= -5.0\ \text{m/s}$$

$$= 5.0\ \text{m/s, west}$$

Textbook question 25:



$m_{fb} = 92\ \text{kg}$

$m_{\ell b_1} = m_{\ell b_2} = 75\ \text{kg}$

$\vec{v}_{fb} = +5.0\ \text{m/s}$

$\vec{v}_{\ell b_1} = -2.0\ \text{m/s}$

$\vec{v}_{\ell b_2} = -4.0\ \text{m/s}$

$\vec{v}' = ?$

$$\sum \vec{p}_{before} = \sum \vec{p}_{after}$$

$$m_{fb}\vec{v}_{fb} + m_{\ell b_1}\vec{v}_{\ell b_1} + m_{\ell b_2}\vec{v}_{\ell b_2} = m_{fb}\vec{v}_{fb}{}' + m_{\ell b_1}\vec{v}_{\ell b_1}{}' + m_{\ell b_2}\vec{v}_{\ell b_2}{}'$$

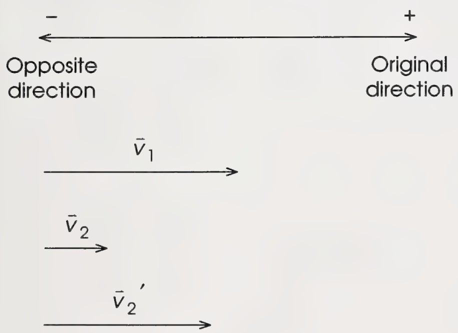
and since $\vec{v}_{fb}{}' = \vec{v}_{\ell b_1}{}' = \vec{v}_{\ell b_2}{}'$, you can write

$$m_{fb}\vec{v}_{fb} + m_{\ell b_1}\vec{v}_{\ell b_1} + m_{\ell b_2}\vec{v}_{\ell b_2} = \left(m_{fb} + m_{\ell b_1} + m_{\ell b_2}\right)\vec{v}\,'$$

$$\begin{aligned}\vec{v}\,' &= \frac{m_{fb}\vec{v}_{fb} + m_{\ell b_1}\vec{v}_{\ell b_1} + m_{\ell b_2}\vec{v}_{\ell b_2}}{m_{fb} + m_{\ell b_1} + m_{\ell b_2}} \\ &= \frac{(92\text{ kg})(+5.0\text{ m/s}) + (75\text{ kg})(-2.0\text{ m/s}) + (75\text{kg})(-4.0\text{ m/s})}{(92\text{ kg}) + (75\text{ kg}) + (75\text{ kg})} \\ &= +0.041\text{ m/s}\end{aligned}$$

The fullback will continue on into the end zone and score.

Textbook question 27:



$$\begin{aligned} m_1 &= 0.200\text{ kg} \\ m_2 &= 0.100\text{ kg} \\ \vec{v}_1 &= +0.30\text{ m/s} \\ \vec{v}_2 &= +0.10\text{ m/s} \\ \vec{v}_2{}' &= +0.26\text{ m/s} \\ \vec{v}_1{}' &= ? \end{aligned}$$

$$\sum \vec{p}_{before} = \sum \vec{p}_{after}$$

$$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1{}' + m_2\vec{v}_2{}'$$

$$\begin{aligned}\vec{v}_1{}' &= \frac{m_1\vec{v}_1 + m_2\vec{v}_2 - m_2\vec{v}_2{}'}{m_1} \\ &= \frac{(0.200\text{ kg})(+0.30\text{ m/s}) + (0.100\text{ kg})(0.10\text{m/s}) - (0.100\text{ kg})(+0.26\text{ m/s})}{0.200\text{ kg}} \\ &= +0.22\text{ m/s} \\ &= 0.22\text{ m/s in the original direction}\end{aligned}$$

9. Textbook question 6:

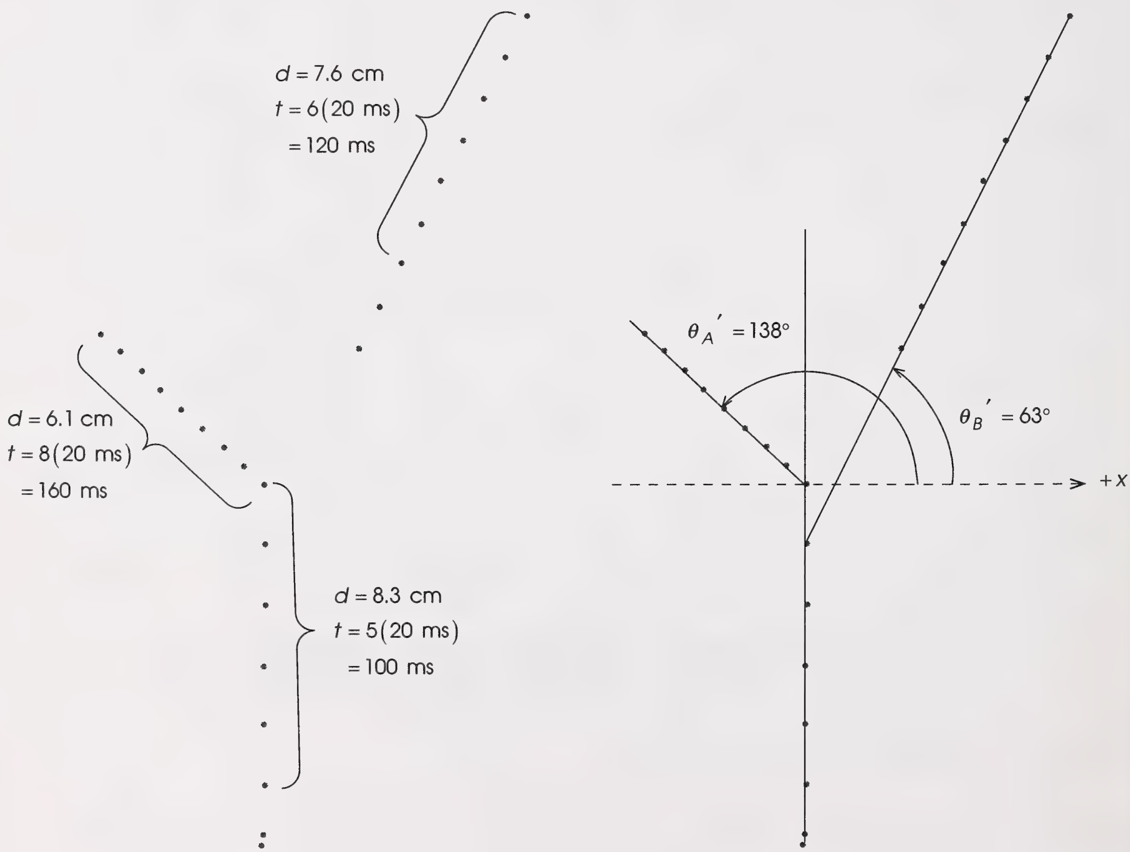
   The momentum of the tennis ball has obviously been changed. Therefore, the tennis ball alone is not an isolated system. However, if you consider the system to be the ball, the wall, and the earth, this collision will follow the law of conservation of momentum. The change in momentum of the tennis ball will be balanced by an equal change in the momentum of the wall and the earth, but in the opposite direction. Therefore, the wall and Earth would have their velocities changed, however small that change might be.

   Textbook question 9:

   If the rifle is held loosely, the recoil momentum works on the rifle only. With the rifle held firmly against your shoulder, the recoil momentum works on you and the rifle. Since the rifle alone is much less massive, it will have a larger velocity and painfully strike your shoulder.

Section 2: Activity 4

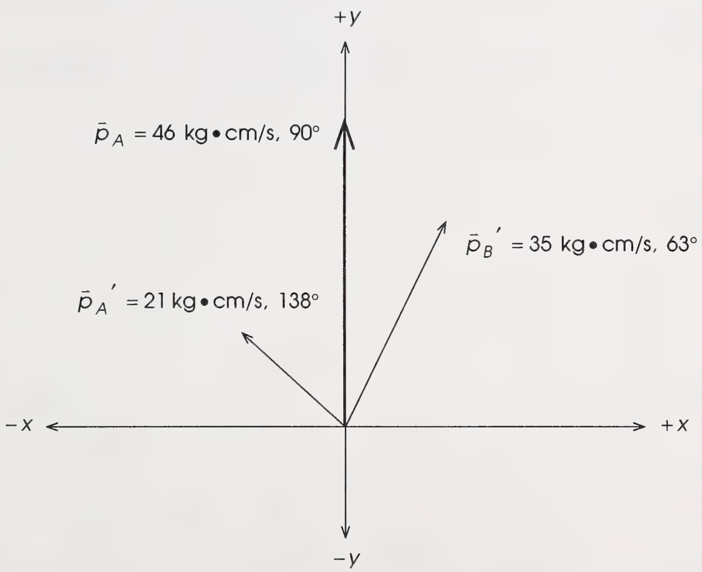
1. The following data was taken from the pull-out page. These diagrams are not drawn to scale. If you produced your own air-table read out, your values will differ, but the technique shown will still apply.



	Before Collision		After Collision	
	Puck A	Puck B	Puck A	Puck B
Mass (kg)	0.550	0.550	0.550	0.550
Distance (cm)	8.3	0	6.1	7.6
Time (s)	0.100	–	0.160	0.120
Magnitude of Velocity (cm/s)	83	0	38	63
Momentum (kg • cm/s)	46	0	21	35
Angle to Indicate Direction (°)	90	–	138°	63°

The figures for speed and momentum are rounded to two digits. The rounded values are used in the calculations that follow.

2. The following sample answer matches the data taken from the tear-out sheet.



3. The following sample calculations matches the data taken from the pull-out page.

- Resolve each of the momentum vectors into x- and y-components.

$$\begin{aligned} p_{A_x}{}' &= p_A{}'(\cos\theta) \\ &= (21\ \text{kg} \bullet \text{cm/s})(\cos 138°) \\ &= -15.61\ \text{kg} \bullet \text{cm/s} \\ &= -16\ \text{kg} \bullet \text{cm/s} \end{aligned}$$

$$\begin{aligned} p_{A_y}{}' &= p_A{}'(\sin\theta) \\ &= (21\ \text{kg} \bullet \text{cm/s})(\sin 138°) \\ &= 14.05\ \text{kg} \bullet \text{cm/s} \\ &= 14\ \text{kg} \bullet \text{cm/s} \end{aligned}$$

$$\begin{aligned} p_{B_x}{}' &= p_B{}'(\cos\theta) \\ &= (35\ \text{kg} \bullet \text{cm/s})(\cos 63°) \\ &= 15.89\ \text{kg} \bullet \text{cm/s} \\ &= 16\ \text{kg} \bullet \text{cm/s} \end{aligned}$$

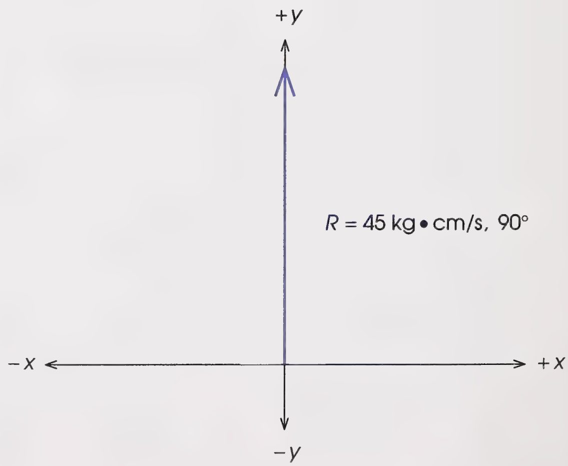
$$\begin{aligned} p_{B_y}{}' &= p_B{}'(\sin\theta) \\ &= (35\ \text{kg} \bullet \text{cm/s})(\sin 63°) \\ &= 31.19\ \text{kg} \bullet \text{cm/s} \\ &= 31\ \text{kg} \bullet \text{cm/s} \end{aligned}$$

- Find the resultant.

$$\begin{aligned} R_x &= p_{A_x}{}' + p_{B_x}{}' \\ &= (-16\ \text{kg} \bullet \text{cm/s}) + (16\ \text{kg} \bullet \text{cm/s}) \\ &= 0\ \text{kg} \bullet \text{cm/s} \end{aligned}$$

$$\begin{aligned} R_y &= p_{A_y}{}' + p_{B_y}{}' \\ &= (14\ \text{kg} \bullet \text{cm/s}) + (31\ \text{kg} \bullet \text{cm/s}) \\ &= 45\ \text{kg} \bullet \text{cm/s} \end{aligned}$$

$$\begin{aligned} C^2 &= A^2 + B^2 \\ R^2 &= R_x{}^2 + R_y{}^2 \\ R &= \sqrt{R_x{}^2 + R_y{}^2} \\ &= \sqrt{(0\ \text{kg} \bullet \text{cm/s})^2 + (45\ \text{kg} \bullet \text{cm/s})^2} \\ &= 45\ \text{kg} \bullet \text{cm/s} \end{aligned}$$



The total of the momenta after the collision is $45\ \text{kg} \bullet \text{cm/s}, 90°$.

4. The following answer matches the sample data presented earlier.

The total of the momenta after the collision is 45 kg • cm/s, 90°, while the total of the momenta before the collision is 46 kg • cm/s, 90°. The fact that these values are so close provides strong evidence that momentum is conserved in two dimensions.

If you performed the analysis using your own data, you should be able to reach the same conclusion regarding the initial momentum and the resultant, $\vec{R}$. The two should be very close if the procedure was correctly followed. The two x-components of the final momenta should nearly cancel each other, leaving the resultant with an angle of almost 90°. The two y-components should have a sum almost equal to the initial momentum of Puck A.

Section 2: Activity 5

1. This vector sum method is possible in almost any question. However, when none of the angles are 90°, the method becomes impossible without knowledge of the law of cosines or the sine law. It is at this time that components are best used.

2. Textbook question 13:

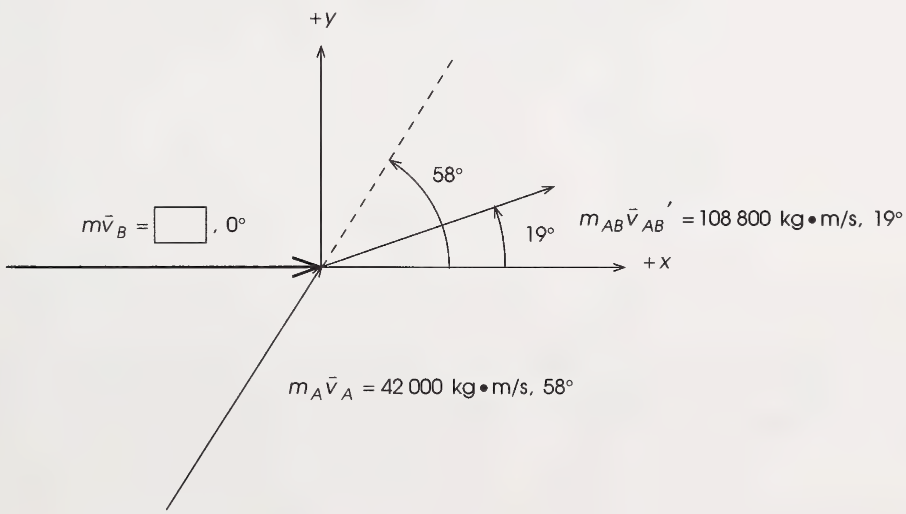
   The solution can be found on page 670 of your textbook.

   Textbook question 14:

   The solution can be found on page 671 of your textbook.

3. You must first make the general assumption that momentum is conserved in this collision. Proceed by calculating each known momentum and finding the necessary components.

   • Redraw the initial diagram in terms of momentum and the x-y axis.



- From the initial diagram, it can be seen that the momentum of car B is entirely along the x-axis. Therefore, the velocity of car B must also be a vector with only an x-component. This means that the problem can be solved by looking at x-components only.

- Apply the law of conservation of momentum.

$$\boxed{\sum \bar{p}_{\text{before}} = \sum \bar{p}_{\text{after}}}$$

Since the components are also conserved, the law can be rewritten as follows.

$$\sum (p_x)_{\text{before}} = \sum (p_x)_{\text{after}} \qquad \sum (p_y)_{\text{before}} = \sum (p_y)_{\text{after}}$$

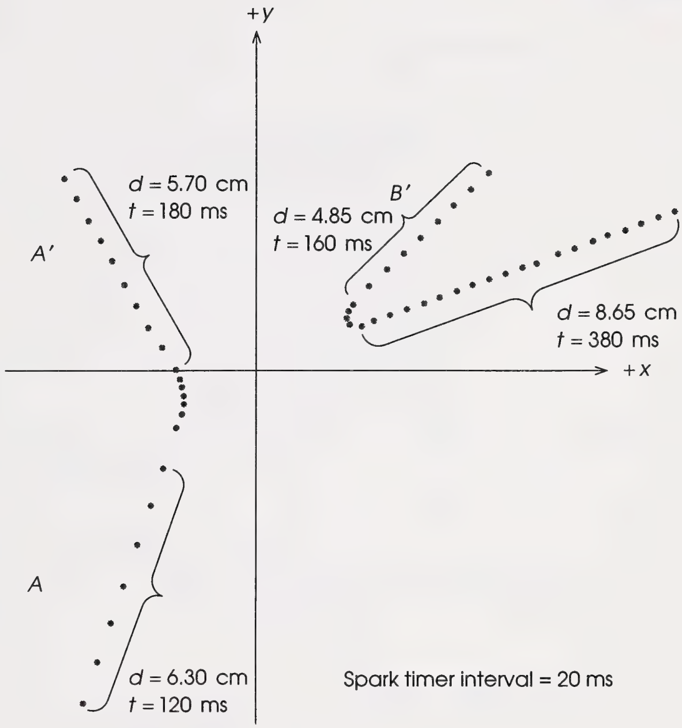
- Use the x-components to solve for the magnitude of the velocity of car B.

$$\begin{aligned}
\sum p_{x\,\text{before}} &= \sum p_{x\,\text{after}} \\
m_B v_B + (m_A v_A)_x &= (m_{AB} v_{AB}{}')_x \\
m_B v_B &= (m_{AB} v_{AB}{}')_x - (m_A v_A)_x \\
v_B &= \frac{(m_{AB} v_{AB}{}')_x - (m_A v_A)_x}{m_B} \\
&= \frac{(108\,800 \text{ kg}\bullet\text{m/s})(\cos 19°) - (42\,000 \text{ kg}\bullet\text{m/s})(\cos 58°)}{2000 \text{ kg}} \\
&= \frac{(102\,872 \text{ kg}\bullet\text{m/s}) - (22\,257 \text{ kg}\bullet\text{m/s})}{2000 \text{ kg}} \\
&= \frac{80\,615 \text{ kg}\bullet\text{m/s}}{2000 \text{ kg}} \\
&= 40.3 \text{ m/s} \\
&= 40 \text{ m/s} \\
&= 145 \text{ km/h}
\end{aligned}$$

- The velocity of car B has a magnitude of 40 m/s, or 145 km/h, so it was exceeding the speed limit.

4. The following diagrams show sample measurements. Yours may differ for distance and time, but the angle measurements should be close to the listed values. The speed and momentum calculations are shown as well, and should agree closely with your values. From that point of the solution onward, your calculations should be similar.

$v = \frac{d}{t}$, $p = mv$



A $\quad v_A = \dfrac{6.30 \text{ cm}}{0.120 \text{ s}}$

$= 52.5 \text{ cm/s}$

$p_A = (0.715 \text{ kg})(52.5 \text{ cm/s})$

$= 37.5 \text{ kg} \bullet \text{cm/s}$

B $\quad v_B = \dfrac{8.65 \text{ cm}}{0.380 \text{ s}}$

$= 22.8 \text{ cm/s}$

$p_B = (0.550 \text{ kg})(22.8 \text{ cm/s})$

$= 12.5 \text{ kg} \bullet \text{cm/s}$

A′ $\quad v_A' = \dfrac{5.70 \text{ cm}}{0.180 \text{ s}}$

$= 31.7 \text{ cm/s}$

$p_A' = (0.715 \text{ kg})(31.7 \text{ cm/s})$

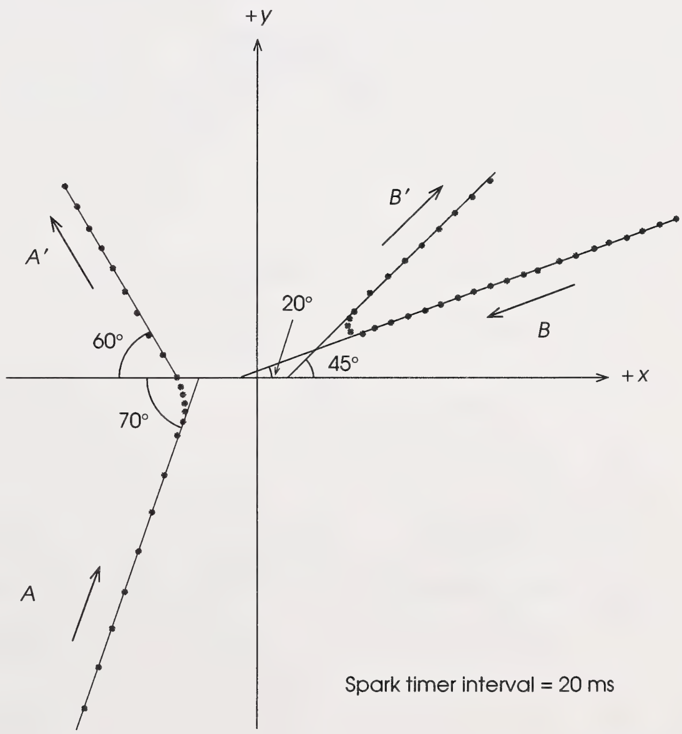
$= 22.6 \text{ kg} \bullet \text{cm/s}$

B′ $\quad v_B' = \dfrac{4.85 \text{ cm}}{0.160 \text{ s}}$

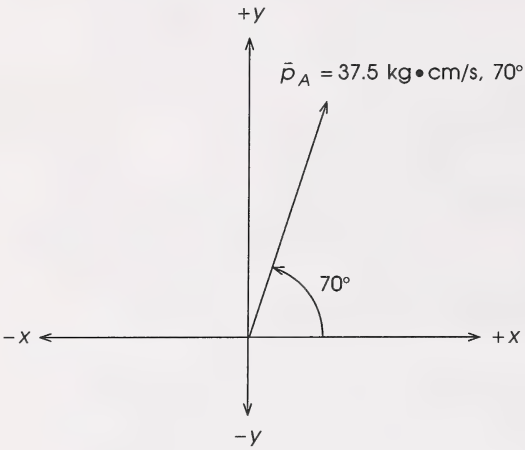
$= 30.3 \text{ cm/s}$

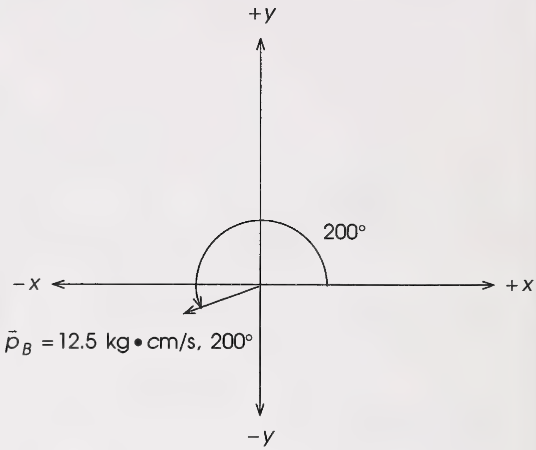
$p_B' = (0.550 \text{ kg})(30.3 \text{ cm/s})$

$= 16.7 \text{ kg} \bullet \text{cm/s}$

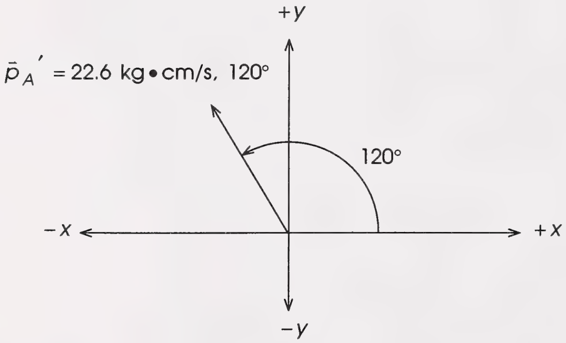


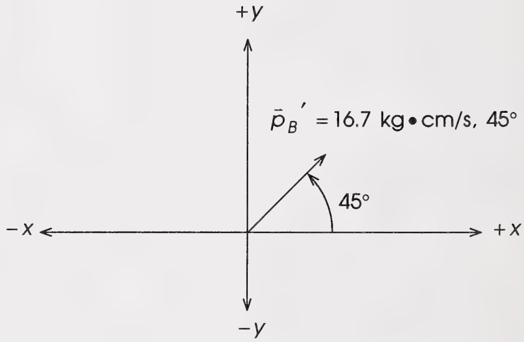
- Before the collision:





- After the collision:





Now you must determine the components.

$\vec{p}_A = 37.5 \text{ kg}\bullet\text{cm/s, } 70°$

$$p_{A_x} = (37.5 \text{ kg}\bullet\text{cm/s})(\cos 70°) = 12.8 \text{ kg}\bullet\text{cm/s}$$

$$p_{A_y} = (37.5 \text{ kg}\bullet\text{cm/s})(\sin 70°) = 35.2 \text{ kg}\bullet\text{cm/s}$$

$\vec{p}_B = 12.5 \text{ kg}\bullet\text{cm/s, } 200°$

$$p_{B_x} = (12.5 \text{ kg}\bullet\text{cm/s})(\cos 200°) = -11.7 \text{ kg}\bullet\text{cm/s}$$

$$p_{B_y} = (12.5 \text{ kg}\bullet\text{cm/s})(\sin 200°) = -4.3 \text{ kg}\bullet\text{cm/s}$$

$\vec{p}_A{}' = 22.6\ \text{kg} \bullet \text{cm/s},\ 120°$

$\vec{p}_{A_x}{}' = (22.6\ \text{kg} \bullet \text{cm/s})(\cos\ 120°)$
$= -11.3\ \text{kg} \bullet \text{cm/s}$

$\vec{p}_{A_y}{}' = (22.6\ \text{kg} \bullet \text{cm/s})(\sin\ 120°)$
$= 19.6\ \text{kg} \bullet \text{cm/s}$

$\vec{p}_B{}' = 16.7\ \text{kg} \bullet \text{cm/s},\ 45°$

$\vec{p}_{B_x}{}' = (16.7\ \text{kg} \bullet \text{cm/s})(\cos\ 45°)$
$= 11.8\ \text{kg} \bullet \text{cm/s}$

$\vec{p}_{B_y}{}' = (16.7\ \text{kg} \bullet \text{cm/s})(\sin\ 45°)$
$= 11.8\ \text{kg} \bullet \text{cm/s}$

To find the resultant initial momentum, determine R_x and R_y.

$R_x = p_{A_x} + p_{B_x}$
$= (12.8\ \text{kg} \bullet \text{cm/s}) + (-11.7\ \text{kg} \bullet \text{cm/s})$
$= 1.1\ \text{kg} \bullet \text{cm/s}$

$R_y = p_{A_y} + p_{B_y}$
$= (35.2\ \text{kg} \bullet \text{cm/s}) + (-4.3\ \text{kg} \bullet \text{cm/s})$
$= 30.9\ \text{kg} \bullet \text{cm/s}$

$R = \sqrt{R_x{}^2 + R_y{}^2}$
$= \sqrt{(1.1\ \text{kg} \bullet \text{cm/s})^2 + (30.9\ \text{kg} \bullet \text{cm/s})^2}$
$= 30.9\ \text{kg} \bullet \text{cm/s}$

$\tan\ \theta' = \dfrac{R_x}{R_y}$
$= \dfrac{1.1\ \text{kg} \bullet \text{cm/s}}{30.9\ \text{kg} \bullet \text{cm/s}}$
$\theta' = 2.0°$
$\theta = 88°$

The resultant initial momentum is $30.9\ \text{kg} \bullet \text{cm/s},\ 88°$.

Determine the resultant final momentum.

$R_x{}' = p_{A_x}{}' + p_{B_x}{}'$
$= (-11.3\ \text{kg} \bullet \text{cm/s}) + (11.8\ \text{kg} \bullet \text{cm/s})$
$= 0.5\ \text{kg} \bullet \text{cm/s}$

$R_y{}' = p_{A_y}{}' + p_{B_y}{}'$
$= (19.6\ \text{kg} \bullet \text{cm/s}) + (11.8\ \text{kg} \bullet \text{cm/s})$
$= 31.4\ \text{kg} \bullet \text{cm/s}$

$R' = \sqrt{R_x{}'^2 + R_y{}'^2}$
$= \sqrt{(0.5\ \text{kg} \bullet \text{cm/s})^2 + (31.4\ \text{kg} \bullet \text{cm/s})^2}$
$= 31.4\ \text{kg} \bullet \text{cm/s}$

$\tan\ \theta' = \dfrac{R_x{}'}{R_y{}'}$
$= \dfrac{0.5\ \text{kg} \bullet \text{cm/s}}{30.9\ \text{kg} \bullet \text{cm/s}}$
$\theta' = 0.9°$
$\theta = 89.1°$
$\theta = 89°$

The resultant final momentum is $31.4\ \text{kg} \bullet \text{cm/s},\ 89°$.

Comparing the resultant momentum before with the resultant momentum after indicates that momentum is conserved in this collision.

Section 2: Activity 6

1. In Module 1 you worked with the law of conservation of energy. In Module 2 you have used the law of conservation of momentum.

2. a. Immediately after the collision, the momentum will be conserved in the system of the two cars. However, only seconds after the crash, the cars will come to rest. During this time, the momentum will be transferred to the road and Earth.

   b. Energy is conserved, but will take on many forms. Before the collision, the two cars possess kinetic energy due to their motion. During the crash, this kinetic energy will be transformed into doing work on (bending) the metal, thermal energy, and sound. As well, some of the energy will be converted to light, in the form of sparks. Even immediately after the collision, the kinetic energy will be greatly diminished.

3. In both collisions the momentum will be conserved immediately after the collision. This momentum will eventually be lost to the surface (road or table) and the earth. The major difference is in the energy interaction. The kinetic energy of the two cars is almost immediately transferred totally to doing work on the metal, thermal energy, and sound. However, in the case of the billiard balls, not much thermal energy would be generated by their collision. So, in the first seconds after the billiard ball's collision, very little kinetic energy would be lost.

4. In an inelastic collision, the kinetic energy is converted into other forms of energy. In such a collision you might see permanent deformation of the colliding bodies due to work being done or large amounts of thermal energy and sound created by the collision. In some situations light may also be emitted.

5. The colliding bodies will stick together in a perfectly inelastic collision. A good example would be the collision between two putty balls colliding and sticking together.

6. Textbook question 14. a.:

   If the ball returned to its original height, no loss of kinetic energy occurred during the collision. In this case it must be a perfectly elastic collision.

   Textbook question 14. b.:

   If the ball did not bounce, all of the kinetic energy was lost and the collision was totally inelastic.

   Textbook question 14. c.:

   The energy lost by the ball was transformed into thermal energy, sound, and the work done during the deformation of the ball while it was touching the floor.

7. a. $\sum E_{k\,before} = E_{k\,A} + E_{k\,B}$

$= \frac{1}{2}m_A v_A^{\,2} + 0$

$= \frac{1}{2}(0.550\ \text{kg})(0.65\ \text{m/s})^2$

$= 0.12\ \text{J}$

$\sum E_{k\,after} = E_{k\,A}' + E_{k\,B}'$

$= \frac{1}{2}m_A v_A'^{\,2} + \frac{1}{2}m_B v_B'^{\,2}$ $\quad (v_A' = v_B' = v')$

$= \frac{1}{2}(m_A + m_B)v'^{\,2}$ $\quad (m_A = m_B = 0.550\ \text{kg})$

$= \frac{1}{2}(1.100\ \text{kg})(0.31\ \text{m/s})^2$

$= 0.053\ \text{J}$

b. loss of $E_k = 0.12\ \text{J} - 0.053\ \text{J}$

$= 0.067\ \text{J}$

$\text{percent loss} = \frac{0.067\ \text{J}}{0.12\ \text{J}} \times 100\%$

$= 56\%$

c. This is an inelastic collision because the objects stick together. Over half of the original E_k was lost in the collision. To classify it as highly elastic, it would need to be closer to 75% or 90% conserved.

8. a. $\sum E_{k\,before} = E_{k\,A} + E_{k\,B}$

$= \frac{1}{2}m_A v_A^{\,2} + 0$

$= \frac{1}{2}(0.550\ \text{kg})(0.83\ \text{m/s})^2$

$= 0.19\ \text{J}$

$\sum E_{k\,after} = E_{k\,A}' + E_{k\,B}'$

$= \frac{1}{2}m_A v_A'^{\,2} + \frac{1}{2}m_B v_B'^{\,2}$ $\quad (m_A = m_B = m)$

$= \frac{1}{2}m\left(v_A'^{\,2} + v_A'^{\,2}\right)$

$= \frac{1}{2}(0.550\ \text{kg})\left[(0.38\ \text{m/s})^2 + (0.63\ \text{m/s})^2\right]$

$= 0.15\ \text{J}$

b. loss of $E_k = 0.19\ \text{J} - 0.15\ \text{J}$

$= 0.04\ \text{J}$

$\text{percent loss} = \frac{0.04\ \text{J}}{0.19\ \text{J}} \times 100\%$

$= 21\%$

c. This collision is much more elastic than the previous one. In fact, since the energy is 79% conserved, you could classify it as highly elastic.

9. Textbook question 13:

The solution can be found on page 673 of your textbook.

10. Begin with the third statement in the solution.

$$\frac{1}{2}mv^2 = \frac{1}{2}mv'^2 + \frac{1}{2}M\left[\frac{m(v-v')}{M}\right]^2$$

Multiply by $\frac{2}{m}$. →
$$v^2 = v'^2 + \frac{M}{m}\left[\frac{m^2(v-v')^2}{M^2}\right]$$

$$v^2 - v'^2 = \frac{m}{M}(v-v')^2$$

Factor. →
$$(v+v')(v-v') = \frac{m}{M}(v-v')(v-v')$$

Divide by $v-v'$. →
$$v+v' = \frac{m}{M}(v-v')$$

Multiply by M and isolate v'. →
$$Mv + Mv' = mv - mv'$$

$$mv' + Mv' = mv - Mv$$

$$v'(m+M) = v(m-M)$$

Solve for v'. →
$$v' = \frac{m-M}{m+M}v$$

The derivation of $V' = \frac{2mv}{m+M}$ begins with the following initial equation:

$$V' = m\left(\frac{v-v'}{M}\right)$$

Substitution for v'. →
$$V' = m\left[\frac{v - \left(\frac{m-M}{m+M}\right)v}{M}\right]$$

$$V' = \frac{m}{M}\left[v - \left(\frac{m-M}{m+M}\right)v\right]$$

$$V' = \frac{m}{M}\left[\frac{v(m+M) - v(m-M)}{m+M}\right]$$

$$V' = \frac{m}{M}\left[\frac{mv + Mv - mv + Mv}{m+M}\right]$$

$$V' = \frac{m}{M}\left(\frac{2Mv}{m+M}\right)$$

$$V' = \frac{2mv}{m+M}$$

11. Textbook question 15:

The solution can be found on pages 673 and 674 of your textbook.

12.

$-$ $\longleftrightarrow$ $+$

Opposite direction | Forwards

mass of bullet $(m_A) = 4.85 \times 10^{-3}$ kg

velocity of bullet $(\vec{v}_A) = +214$ m/s

$\longrightarrow$

mass of block $(m_B) = 3.219$ kg

$$\sum \vec{p}_{before} = \sum \vec{p}_{after}$$

$$m_A\vec{v}_A + m_B\vec{v}_B = (m_A + m_B)\vec{v}' \qquad (\vec{v}_B = 0)$$

$$m_A\vec{v}_A + 0 = (m_A + m_B)\vec{v}'$$

$$\vec{v}' = \frac{m_A\vec{v}_A}{m_A + m_B}$$

$$= \frac{(4.85 \times 10^{-3}\ \text{kg})(+214\ \text{m/s})}{(4.85 \times 10^{-3}\ \text{kg}) + (3.219\ \text{kg})}$$

$$= +0.322\ \text{m/s}$$

The bullet and block move together at 0.322 m/s forwards.

$$\sum E_{k\,before} = \frac{1}{2}m_A v_A^{\,2} + \frac{1}{2}m_B v_B^{\,2} \qquad (v_B = 0)$$

$$= \frac{1}{2}(4.85 \times 10^{-3}\ \text{kg})(214\ \text{m/s})^2 + 0$$

$$= 111\ \text{J}$$

$$\sum E_{k\,after} = \frac{1}{2}m_A v_A'^{\,2} + \frac{1}{2}m_B v_B'^{\,2} \qquad (v_A' = v_B' = v')$$

$$= \frac{1}{2}(m_A + m_B)v'^{\,2}$$

$$= \frac{1}{2}(4.85 \times 10^{-3}\ \text{kg} + 3.219\ \text{kg})(0.322\ \text{m/s})^2$$

$$= 0.167\ \text{J}$$

All but a very small amount of the E_k is lost.

13.

$-$ $\longleftrightarrow$ $+$

Opposite direction | Forwards

$m_A = 0.510$ kg $\qquad$ $m_B = 0.675$ kg

$\longrightarrow$

$\vec{v}_A = +40$ cm/s $\qquad$ $\vec{v}_B = 0$ cm/s

Use the derived formulae from question 10 to solve for $\vec{v}_A{}'$ and $\vec{v}_B{}'$.

$$\vec{v}_A{}' = \frac{m_A - m_B}{m_A + m_B}\vec{v}_A$$

$$= \frac{(0.510\ \text{kg}) - (0.675\ \text{kg})}{(0.675\ \text{kg}) + (0.675\ \text{kg})}(+40\ \text{cm/s})$$

$$= -5.6\ \text{cm/s}$$

$$\vec{v}_B{}' = \frac{2m_A\vec{v}_A}{m_A + m_B}$$

$$= \frac{2(0.510\ \text{kg})(+40\ \text{cm/s})}{(0.510\ \text{kg}) + (0.675\ \text{kg})}$$

$$= +34\ \text{cm/s}$$

Puck A rebounds in the opposite direction at 5.6 cm/s and puck B moves forwards at 34 cm/s.

Section 2: Follow-up Activities

Extra Help

1. a. Hit and rebound is $m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}_A{'} + m_B\vec{v}_B{'}$.

   Hit and stick is $m_A\vec{v}_A + m_B\vec{v}_B = (m_A + m_B)\vec{v}{'}$.

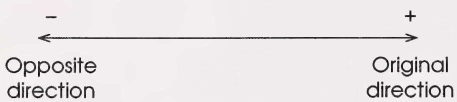
   Explosion is $(m_A + m_B)\vec{v} = m_A\vec{v}_A{'} + m_B\vec{v}_B{'}$. Since the objects are often at rest beforehand, this equation frequently simplifies to $0 = m_A\vec{v}_A{'} + m_B\vec{v}_B{'}$.

   The hit-and-rebound equation shows four separate momenta and follows directly from the law of conservation of momentum, $\vec{p}_A + \vec{p}_B = \vec{p}_A{'} + \vec{p}_B{'}$. The hit-and-stick equation shows a resulting mass equal to the sum of m_A and m_B. Finally, the explosion equation shows that before the explosion, the total momentum is zero.

   b. The basic equation is $m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}_A{'} + m_B\vec{v}_B{'}$.

   c. This is best described as an explosion since the woman and cart are together before she jumped. However, since there was motion before she jumped, the best equation is $(m_A + m_B)\vec{v} = m_A\vec{v}_A{'} + m_B\vec{v}_B{'}$. You will find that the solution (from Activity 3) uses an equation in which the two initial masses are combined.

2. Textbook question 17:

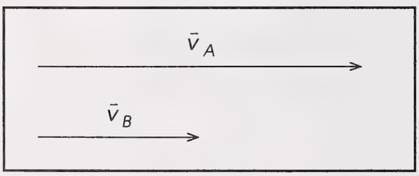


$m_A = 5.0$ g | $m_B = 10.0$ g

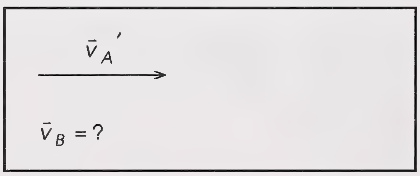
$\vec{v}_A = +20.0$ cm/s | $\vec{v}_B = +10.0$ cm/s

$\vec{v}_A{'} = +8.0$ cm/s | $\vec{v}_B{'} = ?$

Before Collision



After Collision



Textbook question 17.a.:

$$\begin{aligned}\vec{p}_A &= m_A \vec{v}_A \\ &= (5.0\ \text{g})(20.0\ \text{cm/s}) \\ &= 1.0 \times 10^2\ \text{g} \bullet \text{cm/s}\end{aligned}$$

Textbook question 17.b.:

$$\begin{aligned}\Delta\vec{p}_A &= \vec{p}_A{}' - \vec{p}_A \\ \Delta\vec{p}_A &= m_A \vec{v}_A{}' - \vec{p}_A \\ &= (5.0\ \text{g})(8.0\ \text{cm/s}) - (1.0 \times 10^2\ \text{g} \bullet \text{cm/s}) \\ &= -60\ \text{g} \bullet \text{cm/s}\end{aligned}$$

Textbook question 17.c.:

$$\begin{aligned}\Delta\vec{p}_B &= -\Delta\vec{p}_A \\ &= -(-60\ \text{g} \bullet \text{cm/s}) \\ &= 60\ \text{g} \bullet \text{cm/s}\end{aligned}$$

Textbook question 17.d.:

$$\begin{aligned}\Delta\vec{p}_B &= \vec{p}_B{}' - \vec{p}_B \\ \vec{p}_B{}' &= \vec{p}_B + \Delta\vec{p}_B \\ \vec{p}_B{}' &= m_B \vec{v}_B + \Delta\vec{p}_B \\ &= (10.0\ \text{g})(10.0\ \text{cm/s}) + (60\ \text{g} \bullet \text{cm/s}) \\ &= 160\ \text{g} \bullet \text{cm/s}\end{aligned}$$

Textbook question 17.e.:

$$\begin{aligned}\vec{p}_B{}' &= m_B \vec{v}_B{}' \\ \vec{v}_B{}' &= \frac{\vec{p}_B{}'}{m_B} \\ &= \frac{160\ \text{g} \bullet \text{cm/s}}{10.0\ \text{g}} \\ &= 16.0\ \text{cm/s}\end{aligned}$$

Textbook question 20:

$-$ $\longleftrightarrow$ $+$

Opposite direction — Forward

$$\begin{aligned}m_{bullet} &= m_b \\ &= 15\ \text{g}\end{aligned}$$

$$\begin{aligned}m_{block} &= m_{bl} \\ &= 5085\ \text{g}\end{aligned}$$

$$\begin{aligned}\vec{v}_b &= ? \\ \vec{v}_{bl} &= 0\ \text{m/s} \\ \vec{v}_b{}' &= \vec{v}_{bl}{}' = \vec{v}\,' = 1.0\ \text{m/s}\end{aligned}$$

The bullet and block have a common velocity of 1.0 m/s after the collision.

$\longrightarrow$

$\vec{v}\,' = 1.0\ \text{m/s}$

$$\sum \vec{p}_{before} = \vec{p}_{after}$$

$$m_b\vec{v}_b + m_{bl}\vec{v}_{bl} = m_b\vec{v}\,' + m_{bl}\vec{v}\,'$$

$$m_b\vec{v}_b + m_{bl}\vec{v}_{bl} = \left(m_b + m_{bl}\right)\vec{v}\,'$$

$$\vec{v}_b = \frac{\left(m_b + m_{bl}\right)\vec{v}\,' - m_{bl}\vec{v}_{bl}}{m_b}$$

$$= \frac{(15\text{ g} + 5085\text{ g})(1.0\text{ m/s}) - (5085\text{ g})(0\text{ m/s})}{15\text{ g}}$$

$$= 340\text{ m/s}$$

$$= 3.4 \times 10^2\text{ m/s}$$

3. $$\sum E_{k\,before} = \frac{1}{2}m_A v_A{}^2 + \frac{1}{2}m_B v_B{}^2$$

$$= \frac{1}{2}\left(5.0 \times 10^{-3}\text{ kg}\right)(0.20\text{ m/s})^2 + \frac{1}{2}\left(1.00 \times 10^{-2}\text{ kg}\right)(0.100\text{ m/s})^2$$

$$= 1.5 \times 10^{-4}\text{ J}$$

$$\sum E_{k\,after} = \frac{1}{2}m_A v_A{}'^2 + \frac{1}{2}m_B v_B{}'^2$$

$$= \frac{1}{2}\left(5.0 \times 10^{-3}\text{ kg}\right)(0.080\text{ m/s})^2 + \frac{1}{2}\left(1.00 \times 10^{-2}\text{ kg}\right)(0.160\text{ m/s})^2$$

$$= 1.44 \times 10^{-4}\text{ J}$$

$$= 1.4 \times 10^{-4}\text{ J}$$

$$\text{loss of } E_k = \left(1.5 \times 10^{-4}\text{ J}\right) - \left(1.44 \times 10^{-4}\text{ J}\right)$$

$$= 6.0 \times 10^{-6}\text{ J}$$

$$\text{percent loss} = \frac{6.0 \times 10^{-6}\text{ J}}{1.5 \times 10^{-4}\text{ J}} \times 100\%$$

$$= 4.0\%$$

This collision is highly elastic since only 4.0% of the initial E_k was lost in the collision.

Enrichment

1. $$-\ \longleftrightarrow\ +$$

Opposite direction — Forward

$m_{fist\ and\ arm} = m_f = 6.8\text{ kg}$ $\quad$ $m_{head} = m_h = 5.1\text{ kg}$ $\quad$ $\vec{d}_f = 0.56\text{ m}$

Determine the velocity of the fist before impact by assuming uniform motion.

$$\vec{v}_f = \frac{\vec{d}_f}{t}$$

$$= \frac{+0.56\ \text{m}}{\left(3 \times \frac{1}{30}\ \text{s}\right)}$$

$$= +5.6\ \text{m/s}$$

Now find the velocity of the fist and head after impact.

$\sum \vec{p}_{before} = \sum \vec{p}_{after}$, but, since $\vec{v}_h = 0$ and $\vec{v}_f{}' = \vec{v}_h{}' = \vec{v}\,'$

$$m_f \vec{v}_f = \left(m_f + m_h\right)\vec{v}\,'$$

$$\vec{v}\,' = \frac{m_f v_f}{m_f + m_h}$$

$$= \frac{(6.8\ \text{kg})(+5.6\ \text{m/s})}{6.8\ \text{kg} + 5.1\ \text{kg}}$$

$$= +3.2\ \text{m/s}$$

The final step is to determine the force of the blow.

$$\vec{F}_{fh}\Delta t = m_h \Delta\vec{v}_h$$

$$\vec{F}_{fh} = \frac{m_h\left(\vec{v}_h{}' - \vec{v}_h\right)}{\Delta t}$$

$$= \frac{(5.1\ \text{kg})(+3.2\ \text{m/s} - 0\ \text{m/s})}{\left(\frac{1}{2} \times \frac{1}{30}\ \text{s}\right)}$$

$$= +9.8 \times 10^2\ \text{N}$$

The force of the blow is about 9.8×10^2 N in the same direction as the fist is moving.

2. The six questions asked in the video are covered by the six parts of the accompanying pamphlet.

3.

$$\underset{\text{Opposite direction}}{-} \longleftrightarrow \underset{\text{Forward}}{+}$$

a. $m_h = 5.0\ \text{kg}$

$\vec{v}_h = +10\ \text{m/s}$

$\vec{v}_h{}' = 0\ \text{m/s}$

$\vec{F} = ?$

$$\vec{F}\Delta t = m\Delta\vec{v}$$

$$\vec{F} = \frac{m\Delta\vec{v}}{\Delta t}$$

$$= \frac{(5.0\ \text{kg})\left[0\ \text{m/s} - (+10\ \text{m/s})\right]}{2.0 \times 10^{-2}\ \text{s}}$$

$$= -2500\ \text{N}$$

$$\frac{F}{A} = \frac{2500\ \text{N}}{6.0 \times 10^{-4}\ \text{m}^2}$$

$$= 4.2 \times 10^6\ \text{N/m}^2$$

b. $m_b = 70\ \text{kg}$

$\vec{v}_b = +10\ \text{m/s}$

$\vec{v}_b{}' = 0\ \text{m/s}$

$\vec{F} = ?$

$$\vec{F}\Delta t = m\Delta\vec{v}$$
$$\vec{F} = \frac{m\Delta\vec{v}}{\Delta t}$$
$$\vec{F} = \frac{(70\ \text{kg})(0\ \text{m/s} - 10\ \text{m/s})}{0.50\ \text{s}}$$
$$\vec{F} = -1400\ \text{N}$$

$$\frac{F}{A} = \frac{1400\ \text{N}}{0.10\ \text{m}^2} = 1.4\times10^4\ \text{N/m}^2$$

You will notice that the force per unit area for the unrestrained passenger is about 300 times larger.

c. Seatbelts help to reduce injuries by increasing the time involved for the change in momentum. This has the effect of decreasing the stoppng force. Seatbelts also act to increase the area over which the force acts, which reduces the pressure exerted on the body.

4. Begin by solving for $v_1{}'$ in the momentum expression, $v_1{}' = \frac{m_1v_1 + m_2v_2 - m_2v_2{}'}{m_1}$, and substituting into the kinetic energy expression.

$$\frac{1}{2}m_1v_1{}^2 + \frac{1}{2}m_2v_2{}^2 = \frac{1}{2}m_1\left[\frac{m_1v_1 + m_2v_2 - m_2v_2{}'}{m_1}\right]^2 + \frac{1}{2}m_2v_2{}'^2$$

Multiply by 2 and rearrange the bracket. →
$$m_1v_1{}^2 + m_2v_2{}^2 = m_1\left[v_1 + \frac{m_2}{m_1}\left(v_2 - v_2{}'\right)\right]^2 + m_2v_2{}'^2$$

Square the binomial expression. →
$$m_1v_1{}^2 + m_2v_2{}^2 = m_1\left[v_1^2 + \frac{m_2{}^2}{m_1{}^2}\left(v_2 - v_2{}'\right)^2 + \frac{2m_2}{m_1}v_1\left(v_2 - v_2{}'\right)\right] + m_2v_2{}'^2$$

$$m_1v_1{}^2 + m_2v_2{}^2 = m_1v_1{}^2 + \frac{m_2{}^2}{m_1}\left(v_2 - v_2{}'\right)^2 + 2m_2v_1\left(v_2 - v_2{}'\right) + m_2v_2{}'^2$$

Rearrange and factor. →
$$m_2v_2{}^2 - m_2v_2{}'^2 = \frac{m_2{}^2}{m_1}\left(v_2 - v_2{}'\right)\left(v_2 - v_2{}'\right) + 2m_2v_1\left(v_2 - v_2{}'\right)$$

$$m_2\left(v_2{}^2 - v_2{}'^2\right) = \frac{m_2{}^2}{m_1}\left(v_2 - v_2{}'\right)\left(v_2 - v_2{}'\right) + 2m_2v_1\left(v_2 - v_2{}'\right)$$

$$m_2\left(v_2 - v_2{}'\right)\left(v_2 + v_2{}'\right) = \frac{m_2{}^2}{m_1}\left(v_2 - v_2{}'\right)\left(v_2 - v_2{}'\right) + 2m_2v_1\left(v_2 - v_2{}'\right)$$

Factor out m_2 and $\left(v_2 - v_2{}'\right)$. →
$$v_2 + v_2{}' = \frac{m_2}{m_1}\left(v_2 - v_2{}'\right) + 2v_1$$

Multiply by m_1. →
$$m_1v_2 + m_1v_2{}' = m_2v_2 - m_2v_2{}' + 2m_1v_1$$

Solve for $v_2{}'$. →
$$m_1v_2{}' + m_2v_2{}' = 2m_1v_1 + m_2v_2 - m_1v_2$$

$$\left(m_1 + m_2\right)v_2{}' = 2m_1v_1 + \left(m_2 - m_1\right)v_2$$

$$v_2{}' = \frac{2m_1v_1 + \left(m_2 - m_1\right)v_2}{m_1 + m_2}$$

Another solution for v_2' comes from the factor that was removed, $\left(\text{i.e., } v_2 - v_2'\right)$. If $v_2 - v_2' = 0$, this leads to $v_2' = v_2$. This would only occur if no transfer of energy occurs.

Since $v_1' = v_1 + \frac{m_2}{m_1}\left(v_2 - v_2'\right)$, you can substitute the expression for v_2' and get

$$v_1' = v_1 + \frac{m_2}{m_1}\left[v_2 - \frac{2m_1v_1 + (m_2 - m_1)v_2}{m_1 + m_2}\right].$$

Common denominator → $$v_1' = v_1 + \frac{m_2}{m_1}\left[\frac{(m_1 + m_2)v_2 - 2m_1v_1 - (m_2 - m_1)v_2}{m_1 + m_2}\right]$$

Simplify the bracket. → $$v_1' = v_1 + \frac{m_2}{m_1}\left(\frac{2m_1v_2 - 2m_1v_1}{m_1 + m_2}\right)$$

Factor and simplify. → $$v_1' = v_1 + \frac{m_2}{m_1}\left(\frac{2m_1(v_2 - v_1)}{m_1 + m_2}\right)$$

$$v_1' = v_1 + \frac{2m_2(v_2 - v_1)}{m_1 + m_2}$$

Common denominator → $$v_1' = \frac{(m_1 + m_2)v_1 + 2m_2(v_2 - v_1)}{m_1 + m_2}$$

Simplify. → $$v_1' = \frac{m_1v_1 + m_2v_1 + 2m_2v_2 - 2m_2v_1}{m_1 + m_2}$$

$$v_1' = \frac{2m_2v_2 + m_1v_1 - m_2v_1}{m_1 + m_2}$$

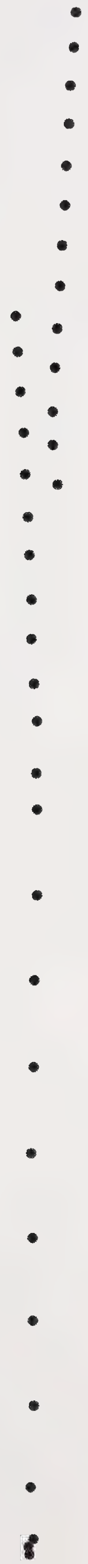
$$v_1' = \frac{2m_2v_2 + (m_1 - m_2)\ v_1}{m_1 + m_2}$$

Of course, if $v_2' = v_2$, $v_1' = v_1$ since no energy is transferred.

Investigation: Conservation of Momentum in One Dimension

This is the sample data for Part B of this investigation.

End



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Beginning

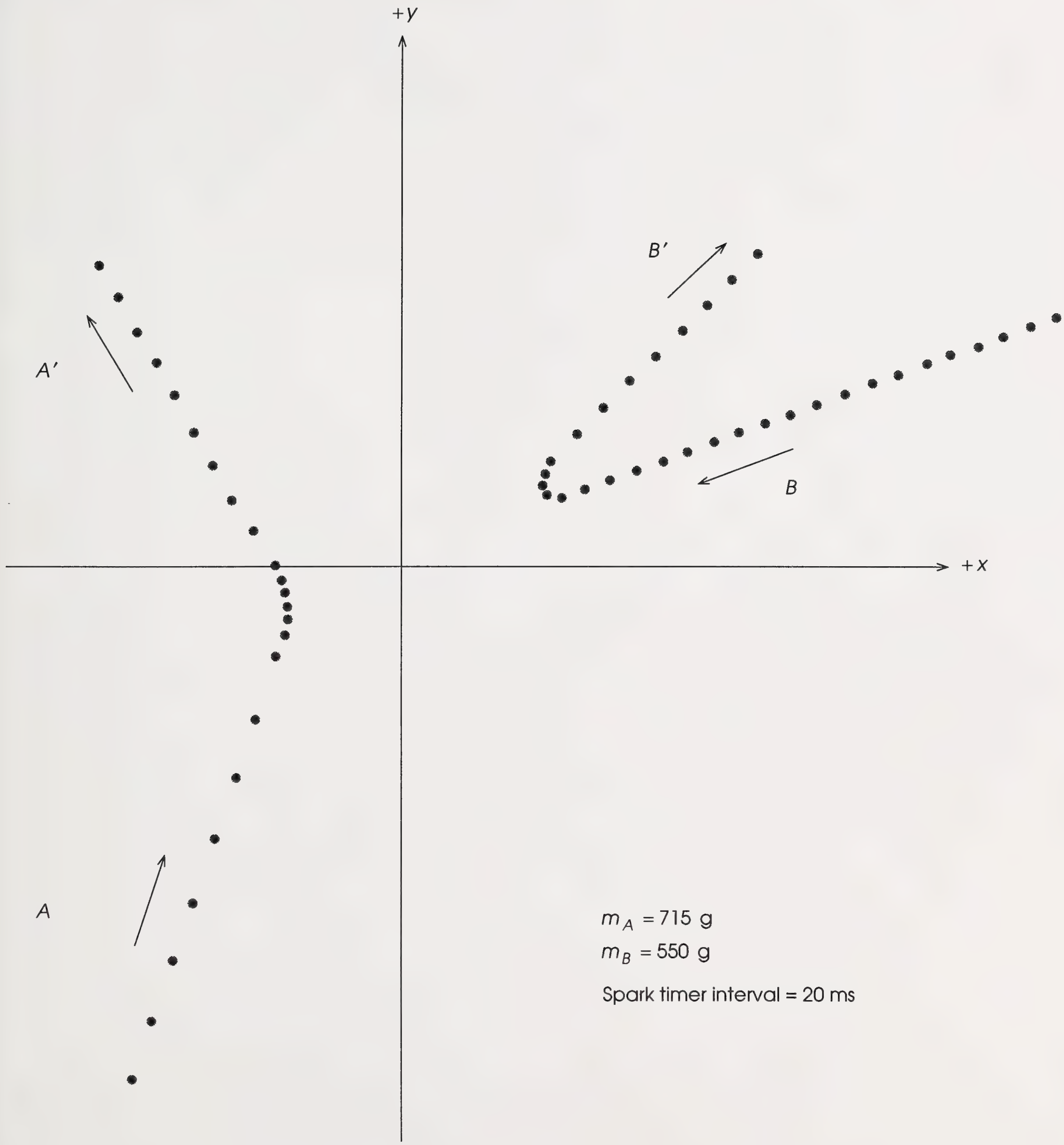
Investigation: A Two-Dimensional Collision

This is the sample data for Part B of this investigation.

End

Section 2: Activity 5: Applications in Two Dimensions

This is the sample data for question 4 of this activity.



NOTES

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Module 2

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